Math 3220 § 2. **Final Exam** Name:____________

Treibergs $\sigma\pi$ May 1, 2018

This is an open book test. You may use the textbook, your notes, homework papers and handouts. No other books, papers, calculators, tablets, laptops, phones or other messaging devices are permitted. Give complete solutions. Be clear about your logic and definitions and justify any theorems that you use. There are [150] total points. **Do SEVEN of eight problems.** If you do more than seven problems, only the first seven will be graded. Cross out the problems you don't wish to be graded.

1.	/21
2.	/21
3.	/21
4.	/21
5.	/22
6.	/22
7.	/22
8.	/22
Total	/135

1. [21] Determine whether the following function $f : \mathbf{R}^2 \to \mathbf{R}$ is differentiable at $(0, 0)$.

$$f(x,y) = \begin{cases} \dfrac{y\sin(x^2+y^2)}{x^2+y^2}, & \text{if } (x,y) \ne (0,0); \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Differentiable at $(0,0)$: ◯	Not differentiable at $(0,0)$: ◯

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Your grades will be posted at my office according to | *Secret Id.* :

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2. (a) [4] Let $A \subseteq \mathbf{R}^d$. Define: A is a *compact set.*

(b) [6] Let $E = \{(x, y) \in \mathbf{R}^2 : x^2 + 2y^2 = 1\}$. Is E compact? Why?

E is compact: ◯ | E is not compact: ◯

(c) [11] For the set E in part (b.), is there a point $x \in E$ closest to $(0, 0)$? Why?

3. Let
$$f(x,y,z,u,v) = x + 2y + 3u - zv,$$
$$g(x,y,z,u,v) = xy + zu + v,$$
$$S = \{(x,y,z,u,v) \in \mathbf{R}^5 : f(x,y,z,u,v) = 2 \text{ and } g(x,y,z,u,v) = 19\}.$$

(a) [4] Define: S is a regular parameterized p-dimensional surface.

(b) [17] Show that S is a regular parameterized p-dimensional surface. Is $S \neq \emptyset$? What is p?

4. Determine whether the statement is true or false. If true give a brief reason. If false, give a counterexample.

   (a) [7] **Statement.** Let $K \subseteq \mathbf{R}^d$ be a compact convex set with nonempty interior and $f \in \mathcal{C}^1(K)$. Then there is $L < \infty$ such that $|f(x) - f(y)| \le L|x - y|$ for all $x, y \in K$.

TRUE: ○	FALSE: ○

   (b) [7] **Statement.** Let $E \subset \mathbf{R}^2$ consist of countably infinitely many distinct points $E = \bigcup_{i=1}^{\infty} \{x_i\}$. Then E is not a Jordan region.

TRUE: ○	FALSE: ○

   (c) [7] **Statement.** Let $R = [0,1]^2$, $f, f_n : R \to \mathbf{R}$ be integrable functions such that $f(x) = \lim_{n\to\infty} f_n(x)$ for all $x \in R$. Then $\int_R f(x)\, dV(x) = \lim_{n\to\infty} \int_R f_n(x)\, dV(x)$.

TRUE: ○	FALSE: ○

5. Let $R \subseteq \mathbf{R}^d$ be an be an aligned rectangle in the in the plane and f be a real valued function on R.

   (a) [6] Define both: f is *integrable* on R and $\displaystyle\int_R f(\mathbf{x})\, dV(\mathbf{x})$.

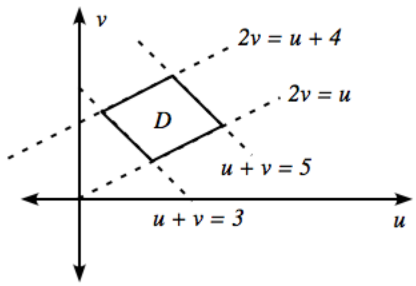
   (b) [6] Complete the statement of a theorem.

   Theorem. Let $R \subset \mathbf{R}^d$ be a rectangle and $f : R \to \mathbf{R}$ be a bounded function. Then f is *integrable* on R if and only if

   (c) [10] Using only the theorem in (b), show that a linear function $f(x) = ax + by$, where a and b are constants, is integrable on $R = [0, 2] \times [0, 3]$.

6. Let $D \subseteq \mathbf{R}^2$ be the region in the first quadrant bounded by the curves $u+v=3$, $u+v=5$, $2v=u$, and $2v=u+4$.

   (a) [11] Find an open set $U \subseteq \mathbf{R}^2$, a one-to-one function $\varphi \in \mathcal{C}^1(U, \mathbf{R}^2)$ such that $\det(\mathrm{d}\varphi(x,y)) \neq 0$ for all $(x,y) \in U$ and an aligned rectangle $R \subseteq U$ such that $D = \varphi(R)$.



   (b) [11] By changing variables using (a), find the integral $\displaystyle\int_D u\,dV(u,v)$.

7. (a) [11] Let $f : \mathbf{R}^2 \to \mathbf{R}^2$ be defined by $f\begin{pmatrix} x \\ \\ y \end{pmatrix} = \begin{pmatrix} x + \sin y \\ \\ \sqrt{1 + x^2 + y^2} \end{pmatrix}$.

Determine whether f is uniformly continuous on $\mathbf{R}^2$ and prove your result.

UNIFORMLY CONTINUOUS: ◯	NOT UNIFORMLY CONTINUOUS: ◯

(b) [11] Let $f : \mathbf{R}^d \to \mathbf{R}$ be continuous. Define what it means for a set E to be *open*. Using just the definition show that $E = \{x \in \mathbf{R}^d : f(x) > 0\}$ is open.

8. (a) [3] Define: $E \subseteq \mathbf{R}^n$ is a *Jordan Region.*

(b) [18] Show that E is a Jordan region, where

$$E = \{(x,y) \in \mathbf{R}^2 : \ (x = 0 \text{ and } 0 \le y \le 1) \text{ or } (y = 0 \text{ and } 0 \le x \le 1) \ \}.$$