Homework for Math 3210 §1, Spring 2015

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April 24, 2015

Our text is by Joseph L. Taylor, *Foundations of Analysis,* American Mathematical Soceity, Providence (2012). The manuscript by Anne Roberts, "Basic Logic Concepts," 2005 is available for download from

$$\texttt{http://www.math.utah.edu/\%7Earoberts/M3210-1d.pdf}$$

Please read the relevant sections in the text as well as any cited reference. Assignments are due the following Friday, or on May 24, whichever comes first.

Your written work reflects your professionalism. Make answers complete and self contained. This means that you should copy or paraphrase each question, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer.

Homework from Wednesday to Tuesday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. The homework reader is (to be determined.) Homework that is placed in his mailbox in JWB 228 before he picks it up at about 4 pm Friday afternoon will be considered to be on time.

Please hand in problems A1 – A5 on Friday, January 16.

A1. Truth table. Construct a truth table for the following statement.

$$[P \wedge (P \Rightarrow Q)] \Rightarrow Q.$$

A2. Equivalent Statements. Verify using truth tables that

$$P \wedge [\sim (Q \wedge R)]$$

is equivalent to

$$(P \wedge [\sim Q]) \vee (P \wedge [\sim R]).$$

A3. Quantified Statements. Determine the truth value of each statement assuming that x, y, z are real numbers.

$$(\exists x)(\forall y)(\exists z)(x + y = z);$$
$$(\exists x)(\forall y)(\forall z)(x + y = z);$$
$$(\forall x)(\forall y)(\exists z)[(z > y) \Rightarrow (z > x + y)];$$
$$(\forall x)(\exists y)(\forall z)[(z > y) \Rightarrow (z > x + y)].$$

A4. Negate and Interpret. Write formally, with quantifiers in the right order. Negate the sentence and interpret.

"Everybody doesn't like something but nobody doesn't like Sara Lee."

A5. Please hand in the following exercises from from Taylor's *Foundations of Analysis*

```
7[2, 5]
```

Please hand in problems B1 – B2 on Friday, Jan. 23.

B1. Functions, Induction and Fields. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
7[9, 12, 13]
15[8, 14, 16]
20[5, 6, 7]
```

B2. Image and intersection. Let A and B be sets and $f : A \to B$ be a function. Show that f is one-to-one if and only if for all subsets $E, F \subset A$,

$$f(E \cap F) = f(E) \cap f(F)$$

Please hand in problem C1 on Friday, Jan. 30.

C1. Ordered Fields and Completeness. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
20[10, 12]
25[2, 4, 6, 8, 9]
```

Please hand in problem D1 on Friday, Feb. 6.

D1. Sup/Inf. Please hand in the following exercises from Taylor's *Foundations of Analysis*

```
31[2, 8a, 9, 12a]
```

Please hand in problem E1–E5 on Friday, Feb. 13.

E1. Limits. Exercises from Taylor's *Foundations of Analysis*

```
38[5]
42[5, 9, 10]
```

E2. Well Ordering The Well Ordering Principle for the natural numbers says that every nonempty subset $S \subset \mathbb{N}$ has a least element. It is a consequence of the Peano axioms (see 15[19]). Show that for every real number x there is an integer $n \in \mathbb{Z}$ such that $n < x \le n + 1$.

E3. Sup/Inf Find the supremum and infimum of the real set E. Prove your result.

$$E = \left\{ \frac{n^2 - 5n + 26}{n^2 - 6n + 10} : n \in \mathbb{N} \right\}.$$

E4. Triangle Inequality Prove that if a, b, x, y are real numbers that satisfy the inequalities

$$|x - a| < 1, \qquad |y - b| < 2, \qquad |a - b| > 7$$

then $|x - y| > 4$.

Please hand in problems F1 on Friday, Feb. 20.

F1. Exercises from Taylor's *Foundations of Analysis.*

```
45[1, 8, 11, 12]
50[3, 4]
```

Please hand in problems G1 – G2 on Friday, Feb. 27.

G1. Exercises from Taylor's *Foundations of Analysis.*

```
50[9, 13]
54[1, 2, 6, 10, 11, 12]
```

G2. Show that the infinite sum exists $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$. [Hint: First prove that $\dfrac{1}{k^2} \leq \dfrac{2}{k} - \dfrac{2}{k+1}$.]

Please hand in problems H1 on Friday, Mar. 6.

H1. Problems from Taylor's *Foundations of Analysis.*

```
64[3, 6, 8, 11]
```

Please hand in problems I1 on Friday, Mar. 13.

I1. Problems from Taylor's *Foundations of Analysis.*

```
68[2, 4, 5, 8, 10, 13]
```

Please hand in problems J1 on Friday, Mar. 27.

J1. Problems from Taylor's *Foundations of Analysis.*

```
73[1, 3, 4, 7, 8]
77[3, 5, 7, 8, 12]
```

Please hand in problems K1 on Friday, Apr. 3.

K1. Problems from Taylor's *Foundations of Analysis.*

```
84[2, 4, 9, 11]
88[1, 9, 11, 12]
92[3, 5, 7, 8]
```

Please hand in problems L1 on Friday, Apr. 10.

L1. Problems from Taylor's *Foundations of Analysis.*

```
98[1, 2, 4, 5, any two from 6-15]
```

Please hand in problems M1 on Friday, Apr. 17.

M1. Problems from Taylor's *Foundations of Analysis.*

```
107[3, 4, 8, 11]
113[3 or 7, 9, 10, 13]
```

Please hand in problems N1 on Friday, Apr. 24.

N1. Problems from Taylor's *Foundations of Analysis.*

```
119[4, 5, 8, 9]
127[1, 9, 13]
133[4, 5, 6, 9, 13]
```

You are responsible for the material in Problem Set O1, but these problems are not to be handed in.

O1. Problems from Taylor's *Foundations of Analysis.*

```
139[1-8, 11]
145[1-6, 10]
```

The FINAL EXAM is Friday, May 1 at 8:00 am.