Math 3160 § 1. **Second Midterm Exam** Name: Sample Problems
Treibergs March 5, 2015

Andrej Cherkaev's Math 3160 Second Midterm Exam given Spring 2000.

1. Find the expansion.
   (a) Find the power series around the point $z_0 = 0$

$$f(z) = \sin z + \frac{1}{z^2+1}$$

   (b) Find the Laurent series in powers of z of the function

$$f(z) = (z-1) + \frac{1}{z-1}$$

2. Calculate the integrals.
   (a) If C is the circle, $|z| = 5$, find

$$I = \int_C \frac{e^z - 1}{\sin z}\, dz$$

   (b) If C is a circle $|z| = 2$, find

$$I = \int_C \frac{\cos z}{z(z^2-8)}\, dz$$

3. Prove the formula

$$\int_{-\infty}^{+\infty} \frac{x \sin ax}{x^4+4}\, dx = \frac{\pi}{2} e^{-a} \sin a.$$

4. (Bonus Problem.) Calculate the integral

$$I = \int_0^{+\infty} \frac{dx}{x^{1/4}(x+4)}\, dx$$