Math 3160 § 1. **Final Exam** Name: Sample Problems
Treibergs March 5, 2015

Andrej Cherkaev's Math 3160 Final Exam given Spring 2000.

1. Solve, write the result in the algebraic and in the exponential forms:
   (a) $(-1)^{\frac{1}{6}} = ?$

   (b) Calculate: $\operatorname{Arg}\left(1 - \dfrac{1}{1+i}\right) = ?$

2. Solve the equations (find all roots).
   (a) $\log z = -1.$

   (b) $\sin z = 6.$

3. Check if
$$u(x,y) = \cos x \cosh y$$
could be the real part of an analytical function $f(x+iy) = u(x,y) + iv(x,y)$. If it could be the real part of an analytical function, find the imaginary part of that function.

4. Expand into the Laurent series if $z_0 = 0$.
$$f = \sin z + \frac{1}{z^2+1}$$

5. Using residues, calculate the improper integral
$$I = \int_{-\infty}^{\infty} \frac{\cos 2x}{x^2 - 2x + 2}\, dx$$

6. Find the distribution of the potential $U(x,y)$ within the unit circle $x^2 + y^2 \le 1$, if the arc $x^2 + y^2 = 1, \quad x \ge 0, y \ge 0$ has zero potential ($U = 0$), and the the rest of the bondary has the potential equal to one ($U = 1$).