MATH 3160-001

EXAM II

Instructions:. There are 4 problems worth 15 points each. Show your work for full credit. Only scientific calculators are allowed. You may use one side of an 8.5 by 11 inch sheet of notes.

1. Let C be the circle of radius 1 centered at i. Compute the contour integral:

$$\int_C \bar{z}^2 \, dz$$

2. Let C be the upper semicircle of radius $R > 1$, centered at the origin, positively oriented (counter clockwise) $x^2 + y^2 = R^2$, $x \geq 0$. Show that

$$|\int_C \frac{\mathrm{Log}(z)}{z^2}\, dz| \leq \frac{(\ell n R + \pi)\pi}{R}$$

3. Let C be the circle of radius 2, centered at the origin, positively oriented. Evaluate the contour integrals:

a.

$$\int_C \frac{sin(z)}{2z+\pi}\, dz$$

b.

$$\int_C \frac{z^4}{(z-i)^3}\, dz$$

4. Suppose that f is an entire function such that $|f(z)| > 1$ for all z. Show that $f(z) =$constant (Hint: Try to apply Liouville's Theorem to an appropriate function). Give an example of an entire function f where $|f(z)| > 0$ for all z, but is not a constant function.