MATH 3160-001

EXAM II

Instructions:. There are 5 problems worth 12 points each. Show your work for full credit. No calculators are allowed. You may use one side of an 8 by 11 inch sheet of notes.

1. Let C be the upper half circle of radius R centered at 0. Show that

$$|\int_C \frac{e^{iz}}{z^2+1}\,dz| \leq \frac{e^{-R}R\pi}{R^2-1}$$

2. Let C be the circle of radius 2, negatively oriented (clockwise), centered at the origin. Evaluate the contour integral:

$$\int_C \frac{e^z}{z^2 - 1}\, dz$$

3. Let C be the circle of radius 1, centered at the origin, positively oriented. Evaluate the contour integral:

$$\int_C \frac{e^z sin(z)}{z^4}\, dz$$

4. Find the MacLaurin series (Taylor series at $z_0 = 0$) for the following function, as well as it's radius of convergence:

$$f(z) = \frac{z}{1 + 4z^2}$$

5. Let $u = u(x, y)$ be a harmonic function on the plane which is the real part of an entire function f. Suppose that $u(x, y) \geq -10$ for all (x, y). Show that $u(x, y) = constant$. (Hint: Try to apply Liouville's Theorem to an appropriate function).