Math 3080 § 1.	**Truck Example:**	Name: Example
Treibergs	**Simple Regression**	Feb. 15, 2014

Simple regression fits the best linear function through observed points in the plane. Today's data was obtained in J. Yanowitz', PhD Thesis "In-Use Emission of Heavy-Duty Diesel Vehicles," Colorado School of Mines 2001 as quoted by Navidi, *Statistics for Engineers and Scientists,* 2nd ed., McGraw Hill, 2008.

It is assumed that the response variable Y, Mileage in this case, is normally distributed with a constant variance σ^2 and with a mean that depends linearly on the explanatory variable x, Weight in this case,

$$y = \beta_0 + \beta_1 x + \epsilon \qquad \text{where } \epsilon \sim N(0, \sigma^2).$$

Given the observed values $\{(x_i, y_i)\}_{i=1,\ldots,n}$, the best line is the one that minimizes the sum of squared deviations. If the proposed line is $y = a + bx$ then the ith deviation is $y_i - a - bx_i$ and the sum square of deviations is

$$f(a,b) = \sum_{i=1}^{n} (y_i - a - bx_i)^2.$$

This is the sum of convex quadratic functions, so is convex in (a, b) and whose minimum may by determined by setting partial derivatives to zero and solving

$$\begin{aligned} 0 &= \frac{\partial f}{\partial a} = -2 \sum_{i=1}^{n} (y_i - a - bx_i) \\ 0 &= \frac{\partial f}{\partial b} = -2 \sum_{i=1}^{n} (y_i - a - bx_i) x_i \end{aligned} \tag{1}$$

The resulting system of equations is

$$\begin{aligned} na + \left(\sum_{i=1}^{n} x_i \right) b &= \sum_{i=1}^{n} y_i \\ \left(\sum_{i=1}^{n} x_i \right) a + \left(\sum_{i=1}^{n} x_i^2 \right) b &= \sum_{i=1}^{n} x_i y_i \end{aligned}$$

Using Cramer's rule we find the solution $(a, b) = (\widehat{\beta_0}, \widehat{\beta_1})$ given by

$$\widehat{\beta_1} = \frac{\begin{vmatrix} n & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i y_i \end{vmatrix}}{\begin{vmatrix} n & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 \end{vmatrix}} = \frac{n \sum_{i=1}^{n} x_i y_i - \left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2} = \frac{S_{xy}}{S_{xx}}$$

where

$$\begin{aligned} S_{xx} &= \sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} x_i \right)^2 \\ S_{xy} &= \sum_{i=1}^{n} x_i y_i - \frac{1}{n} \left(\sum_{i=1}^{n} x_i \right) \left(\sum_{i=1}^{n} y_i \right) \end{aligned}$$

Note that the system is nonsingular if there are at least two distinct x_i so $S_{xx} \neq 0$. If so, we may solve (1) to get

$$\widehat{\beta_0} = \bar{y} - \widehat{\beta_1}\bar{x}.$$

The *mean response* when $x = x^*$ is the point on the fitted line

$$y^* = \widehat{\beta_0} + \widehat{\beta_1}x^*.$$

The *fitted value* is the mean response in case $x = x_i$, in other words

$$\widehat{y_i} = \widehat{\beta_0} + \widehat{\beta_1}x_i.$$

The *predicted value*, $\hat{y}$, is an estimator for the next observation when $x = x^*$. The variability comes from both the error in the next observation and the variability of the response.

$$\hat{y} = \widehat{\beta_0} + \widehat{\beta_1}x^*.$$

As usual, we write random variables as upper case letters. For example, the coefficients of regression $\widehat{B_i}$ computed from a random sample are random variables.

Lemma 1. *$\widehat{B_0}$ and $\widehat{B_1}$ are unbiased estimators for β_0 and β_1 resp. Moreover, for fixed x^*, let the mean response $Y^* = \widehat{B_0} + \widehat{B_1}x^*$ and the predicted value be $\hat{Y} = \widehat{B_0} + \widehat{B_1}x^*$. We have*

$$E(\widehat{B_0}) = \beta_0 \qquad E(\widehat{B_1}) = \beta_1 \qquad \textit{and} \qquad E(Y^*) = E(\hat{Y}) = \beta_0 + \beta_1 x^*.$$

Proof. We have

$$\begin{aligned} E(\bar{Y}|X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) &= E\left(\frac{1}{n}\sum_{i=1}^{n} Y_i | X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n\right) \\ &= \frac{1}{n}\sum_{i=1}^{n} E\left(Y_i | X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n\right) \\ &= \frac{1}{n}\sum_{i=1}^{n} (\beta_0 + \beta_1 x_i) \\ &= \beta_0 + \beta_1 \bar{x}. \end{aligned}$$

Now, given $X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n$,

$$\begin{aligned} E\left(\widehat{B_1}\right) &= E\left(\frac{\sum_{i=1}^n (x_i - \bar{x})(Y_i - \bar{Y})}{\sum_{j=1}^n (x_j - \bar{x})^2}\right) \\ &= \frac{\sum_{i=1}^n (x_i - \bar{x})}{\sum_{j=1}^n (x_j - \bar{x})^2} E\left(Y_i - \bar{Y}\right) \\ &= \frac{\sum_{i=1}^n (x_i - \bar{x})}{\sum_{j=1}^n (x_j - \bar{x})^2} \left(\beta_0 + \beta_1 x_i - \beta_0 - \beta_1 \bar{x}\right) \\ &= \beta_1 \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{\sum_{j=1}^n (x_j - \bar{x})^2} \\ &= \beta_1 \end{aligned}$$

so

$$E\left(\widehat{B_0}\right) = E\left(\bar{Y} - B_1\bar{x}\right) = \beta_0 + \beta_1\bar{x} - \beta_1\bar{x} = \beta_0$$

and

$$E\left(Y^*\right) = E\left(\widehat{B_0} + \widehat{B_1}x^*\right) = \beta_0 + \beta_1 x^*.$$

□

We compute the variances of the regression coefficients, mean response and predicted value.

Lemma 2. *For fixed x^*, the mean response and predicted value is $Y^* = \hat{Y} = \widehat{B_0} + \widehat{B_1}x^*$. The variances of $\widehat{B_0}$, $\widehat{B_1}$, Y^* and $\hat{Y}$ are given by*

$$V(\widehat{B_0}) = \sigma^2\left(\frac{1}{n} + \frac{\bar{x}^2}{S_{xx}}\right), \qquad V(\widehat{B_1}) = \frac{\sigma^2}{S_{xx}},$$

$$V(Y^*) = \sigma^2\left(\frac{1}{n} + \frac{(\bar{x} - x^*)^2}{S_{xx}}\right) \qquad \textit{and} \qquad V(\hat{Y}) = \sigma^2\left(1 + \frac{1}{n} + \frac{(\bar{x} - x^*)^2}{S_{xx}}\right)$$

Proof. First,

$$\begin{aligned} V(Y_i - \bar{Y}) &= V\left(\frac{n-1}{n}Y_i - \frac{1}{n}\sum_{j\neq i} Y_j\right) \\ &= \frac{(n-1)^2}{n^2}\sigma^2 + \frac{n-1}{n^2}\sigma^2 = \frac{n-1}{n}\sigma^2 \end{aligned}$$

Note that $\sum_{i=1}^n (x_i - \bar{x}) = 0$ so

$$\begin{aligned} V(\widehat{B_1}) &= V\left(\frac{\sum_{i=1}^n (x_i - \bar{x})(Y_i - \bar{Y})}{\sum_{j=1}^n (x_j - \bar{x})^2}\right) \\ &= \frac{1}{\left[\sum_{j=1}^n (x_j - \bar{x})^2\right]^2} V\left(\sum_{i=1}^n (x_i - \bar{x})Y_i - \bar{Y}\sum_{i=1}^n (x_i - \bar{x})\right) \\ &= \frac{1}{\left[\sum_{j=1}^n (x_j - \bar{x})^2\right]^2} V\left(\sum_{i=1}^n (x_i - \bar{x})Y_i\right) \\ &= \frac{\sum_{i=1}^n (x_i - \bar{x})^2 V(Y_i)}{\left[\sum_{j=1}^n (x_j - \bar{x})^2\right]^2} \\ &= \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2}. \end{aligned}$$

Second, since $\mathrm{Cov}(Y_i, Y_j) = 0$ if $i \neq j$,

$$\begin{aligned} \mathrm{Cov}(\bar{Y}, \widehat{B_1}) &= \mathrm{Cov}\left(\bar{Y}, \frac{\sum_{i=1}^n (x_i - \bar{x})(Y_i - \bar{Y})}{\sum_{k=1}^n (x_k - \bar{x})^2}\right) \\ &= \frac{\sum_{i=1}^n (x_i - \bar{x})\,\mathrm{Cov}\left(\bar{Y}, Y_i - \bar{Y}\right)}{\sum_{k=1}^n (x_k - \bar{x})^2} \\ &= \frac{\sum_{i=1}^n (x_i - \bar{x})\,\mathrm{Cov}\left(\frac{1}{n}\sum_{j=1}^n Y_j, \frac{n-1}{n}Y_i - \frac{1}{n}\sum_{k\neq i} Y_k\right)}{\sum_{k=1}^n (x_k - \bar{x})^2} \\ &= \frac{\sum_{i=1}^n (x_i - \bar{x})\left[(n-1)V(Y_i) - \sum_{j\neq i} V(Y_j)\right]}{n^2\sum_{k=1}^n (x_k - \bar{x})^2} \\ &= \sigma^2\frac{\sum_{i=1}^n (x_i - \bar{x})[(n-1) - (n-1)]}{\sum_{k=1}^n (x_k - \bar{x})^2} = 0. \end{aligned}$$

Thus

$$\begin{aligned} V(\widehat{B_0}) &= V(\bar{Y} - \widehat{B_1}\bar{x}) \\ &= V(\bar{Y}) - 2\bar{x}\,\mathrm{Cov}(\bar{Y}, \widehat{B_1}) + \bar{x}^2 V(\widehat{B_1}) \\ &= \sigma^2 \left(\frac{1}{n^2} - 0 + \frac{\bar{x}^2}{S_{xx}} \right). \end{aligned}$$

The variance of the mean response follows:

$$\begin{aligned} V(Y^*) &= V(\widehat{B}_0 + \widehat{B_1}x^*) \\ &= V(\bar{Y} + \widehat{B_1}(x^* - \bar{x})) \\ &= V(\bar{Y}) - 2(x^* - \bar{x})\,\mathrm{Cov}(\bar{Y}, \widehat{B_1}) + (x^* - \bar{x})^2 V(\widehat{B_1}) \\ &= \sigma^2 \left(\frac{1}{n^2} - 0 + \frac{(x^* - \bar{x})^2}{S_{xx}} \right). \end{aligned}$$

Finally, the error in predicting the next point Y_{n+1} at $x_{n+1} = x^*$ with Y^* is the variability of

$$Y_{n+1} - \widehat{B_0} - \widehat{B_1}x^*.$$

But Y_{n+1} is independent of $\{Y_i\}_{i=1,\dots,n}$ so

$$\begin{aligned} V(\hat{Y}) &= V(Y_{n+1} - \widehat{B}_0 - \widehat{B_1}x^*) \\ &= V(Y_{n+1}) + V(\widehat{B}_0 + \widehat{B_1}x^*) \\ &= \sigma^2 + \sigma^2 \left(\frac{1}{n^2} + \frac{(x^* - \bar{x})^2}{S_{xx}} \right). \quad \square \end{aligned}$$

The *sum square error* is

$$SSE = \sum_{i=1}^{n} (y_i - \widehat{y_i})^2$$

with corresponding degrees of freedom $n - 2$ because the parameters in the formula $(\widehat{\beta_0}, \widehat{\beta_1})$ have already used up two degrees of freedom.

Lemma 3. *The sum squared error has a shortcut formula*

$$SSE = \sum_{i=1}^{n} y_i^2 - \widehat{\beta_0} \sum_{i=1}^{n} x_i - \widehat{\beta_1} \sum_{i=1}^{n} x_i y_i.$$

Proof. Observe that

$$\sum_{i=1}^{n} \left[y_i - \widehat{\beta_0} - \widehat{\beta_1}x_i \right] = \sum_{i=1}^{n} \left[y_i - \bar{y} - \widehat{\beta_1}(x_i - \bar{x}) \right] = n\bar{y} - n\bar{y} - \widehat{\beta_1}(n\bar{x} - n\bar{x}) = 0$$

and using this,

$$\begin{aligned}\sum_{i=1}^{n}\left[y_i-\widehat{\beta_0}-\widehat{\beta_1}x_i\right]x_i &= \sum_{i=1}^{n}\left[y_i-\widehat{\beta_0}-\widehat{\beta_1}x_i\right](x_i-\bar{x})\\ &= \sum_{i=1}^{n}\left[y_i-\bar{y}-\widehat{\beta_1}(x_i-\bar{x})\right](x_i-\bar{x})\\ &= \sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})+\widehat{\beta_1}\sum_{i=1}^{n}(x_i-\bar{x})^2\\ &= \sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})+\frac{R_{xy}}{R_{xx}}\sum_{i=1}^{n}(x_i-\bar{x})^2\\ &= \sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})-\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})\\ &= 0.\end{aligned}$$

Then the short cut formula follows

$$\begin{aligned}SSE &= \sum_{i=1}^{n}(y_i-\widehat{y_i})^2\\ &= \sum_{i=1}^{n}(y_i-\widehat{\beta_0}-\widehat{\beta_1}x_i)^2\\ &= \sum_{i=1}^{n}(y_i-\widehat{\beta_0}-\widehat{\beta_1}x_i)y_i-\widehat{\beta_0}\sum_{i=1}^{n}(y_i-\widehat{\beta_0}-\widehat{\beta_1}x_i)-\widehat{\beta_1}\sum_{i=1}^{n}(y_i-\widehat{\beta_0}-\widehat{\beta_1}x_i)x_i\\ &= \sum_{i=1}^{n}y_i^2-\widehat{\beta_0}\sum_{i=1}^{n}y_i-\widehat{\beta_1}\sum_{i=1}^{n}x_iy_i.\end{aligned}$$

□

The *mean square error* is

$$MSE=\frac{SSE}{n-2}.$$

Lemma 4. *The mean squared error is an unbiased estimator for the variance*

$$E(MSE)=\sigma^2.$$

Proof. Given $X_1=x_1, X_2=x_2,\ldots,X_n=x_n$, because

$$E(y_i-\widehat{B_0}-\widehat{B_1}x_i)=\beta_0+\beta_1x_i-\beta_0-\beta_1x_i=0,$$

the identity $E(Z^2)=V(Z)+E^2(Z)$ gives

$$\begin{aligned}E(SSE) &= E\left(\sum_{i=1}^{n}(Y_i-\widehat{Y_i})^2\right)\\ &= E\left(\sum_{i=1}^{n}(Y_i-\widehat{B_0}-\widehat{B_1}x_i)^2\right)\\ &= \sum_{i=1}^{n}E\left((Y_i-\widehat{B_0}-\widehat{B_1}x_i)^2\right)\\ &= \sum_{i=1}^{n}V(Y_i-\hat{Y}-\widehat{B_1}(x_i-\bar{x}))\end{aligned}$$

But

$$V(Y_i - \bar{Y} - \widehat{B_1}(x_i - \bar{x})) = V(Y_i - \bar{Y}) - 2(x_i - \bar{x})\operatorname{Cov}(Y_i - \bar{Y}, \widehat{B_1}) + (x_i - \bar{x})^2 V(\widehat{B_1})$$

Finally, since $\operatorname{Cov}(Y_i, Y_j) = 0$ if $i \neq j$ and $\sum_{i \neq j}(x_j - \bar{x}) = -x_i + \bar{x}$ imply

$$\begin{aligned}
\operatorname{Cov}(Y_i - \bar{Y}, \widehat{B_1}) &= \operatorname{Cov}\left(Y_i - \bar{Y}, \frac{\sum_{j=1}^n (x_j - \bar{x})(Y_j - \bar{Y})}{\sum_{k=1}^n (x_k - \bar{x})^2}\right) \\
&= \frac{(x_i - \bar{x})V(Y_i - \bar{Y}) + \sum_{j \neq i}(x_j - \bar{x})\operatorname{Cov}\left(Y_i - \bar{Y}, Y_j - \bar{Y}\right)}{\sum_{k=1}^n (x_k - \bar{x})^2} \\
&= \frac{(n-1)(x_i - \bar{x})\sigma^2}{n\sum_{k=1}^n (x_k - \bar{x})^2} + \frac{\sum_{j \neq i}(x_j - \bar{x})\operatorname{Cov}\left(\frac{n-1}{n}Y_i - \frac{1}{n}\sum_{k \neq i} Y_k, \frac{n-1}{n}Y_j - \frac{1}{n}\sum_{\ell \neq j} Y_\ell\right)}{\sum_{k=1}^n (x_k - \bar{x})^2} \\
&= \frac{(n-1)(x_i - \bar{x})\sigma^2}{n\sum_{k=1}^n (x_k - \bar{x})^2} + \frac{\sum_{j \neq i}(x_j - \bar{x})\left(-(n-1)V(Y_i) - (n-1)V(Y_j) + \sum_{k \neq i,j} V(Y_k)\right)}{n^2\sum_{k=1}^n (x_k - \bar{x})^2} \\
&= \frac{(n-1)(x_i - \bar{x})\sigma^2}{n\sum_{k=1}^n (x_k - \bar{x})^2} - \frac{\sum_{j \neq i}(x_j - \bar{x})\sigma^2}{n\sum_{k=1}^n (x_k - \bar{x})^2} \\
&= \frac{(n-1)(x_i - \bar{x})\sigma^2}{n\sum_{k=1}^n (x_k - \bar{x})^2} + \frac{(x_i - \bar{x})\sigma^2}{n\sum_{k=1}^n (x_k - \bar{x})^2} \\
&= \frac{(x_i - \bar{x})\sigma^2}{\sum_{k=1}^n (x_k - \bar{x})^2}
\end{aligned}$$

Inserting yields

$$\begin{aligned}
E(SSE) &= \sum_{i=1}^n \left[V(Y_i - \bar{Y}) - 2(x_i - \bar{x})\operatorname{Cov}(Y_i - \bar{Y}, \widehat{B_1}) + (x_i - \bar{x})^2 V(\widehat{B_1})\right] \\
&= \sum_{i=1}^n \left[\frac{n-1}{n} - 2\frac{(x_i - \bar{x})^2}{\sum_{k=1}^n (x_k - \bar{x})^2} + \frac{(x_i - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2}\right]\sigma^2 \\
&= [n-2]\,\sigma^2
\end{aligned}$$

□

These lemmas gives estimated standard errors on the quantities.

$$\begin{aligned}
s &= \sqrt{MSE}, \\
s_{\widehat{\beta_0}} &= s\sqrt{\frac{1}{n^2} + \frac{\bar{x}^2}{S_{xx}}}, \\
s_{\widehat{\beta_1}} &= \frac{s}{\sqrt{S_{xx}}}, \\
s_{Y^*} &= s\sqrt{\frac{1}{n^2} + \frac{(x^* - \bar{x})^2}{S_{xx}}}, \\
s_{\hat{Y}} &= s\sqrt{1 + \frac{1}{n^2} + \frac{(x^* - \bar{x})^2}{S_{xx}}}
\end{aligned}$$

We may also define the *total sum square* which adds squares of deviations from the mean, and the *regression sum square* or *residual sum square* measuring the part of variation accounted for by the model by the formulae

$$SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$$

$$SSR = \widehat{\beta_1}^2 \sum_{i=1}^{n} (x_i - \bar{x})^2 = \frac{S_{xy}^2}{S_{xx}}$$

The total sum square has $n-1$ degrees of freedom and the regression sum square has one degree of freedom.

Lemma 5. *The analysis of variance identity,* also called the *sum of squares identity* holds

$$SST = SSR + SSE.$$

Proof. By the shortcut formula,

$$\begin{aligned}
SST &= \sum_{i=1}^{n} (y_i - \bar{y})^2 \\
&= \sum_{i=1}^{n} y_i^2 - n\bar{y}^2 \\
&= \sum_{i=1}^{n} \left(y_i - \widehat{\beta_0} - \widehat{\beta_1} x_i + \widehat{\beta_0} + \widehat{\beta_1} x_i \right)^2 - n\bar{y}^2 \\
&= \sum_{i=1}^{n} \left(y_i - \widehat{\beta_0} - \widehat{\beta_1} x_i \right)^2 + 2 \sum_{i=1}^{n} (y_i - \widehat{\beta_0} - \widehat{\beta_1} x_i)(\widehat{\beta_0} + \widehat{\beta_1} x_i) + \sum_{i=1}^{n} \left(\widehat{\beta_0} + \widehat{\beta_1} x_i \right)^2 - n\bar{y}^2 \\
&= SSE + 0 + \sum_{i=1}^{n} \left(\bar{y} + \widehat{\beta_1}(x_i - \bar{x}) \right)^2 - n\bar{y}^2 \\
&= SSE + n\bar{y}^2 + 2\widehat{\beta_1}\bar{y} \sum_{i=1}^{n} (x_i - \bar{x}) + \widehat{\beta_1}^2 \sum_{i=1}^{n} (x_i - \bar{x})^2 - n\bar{y}^2 \\
&= SSE + SSR
\end{aligned}$$

□

It follows that the *coefficient of determination,*

$$r^2 = 1 - \frac{SSE}{SST} = \frac{SSR}{SST}$$

which falls between $0 \le r^2 \le 1$ is the fraction of variation accounted for by the model. r is called the *residual standard error.*

Data Set Used in this Analysis :

```
# Math 3080 - 1      Truck Data          Feb. 15, 2014
# Treibergs
#
# The following data was obtained by J. Yanowitz, PhD Thesis "In-Use
# Emission of Heavy-Duty Diesel Vehicles," Colorado School of Mines 2001 as
# quoted by Navidi, Statistics for Engineers and Scientists, 2nd ed.,
# McGraw Hill, 2008. Inertial weights (in tons) and fuel economy
# (in mi/gal) was measured for a sample of seven diesel trucks.
#
"Weight" "Mileage"
   8.00     7.69
  24.50     4.97
  27.00     4.56
  14.50     6.49
  28.50     4.34
  12.75     6.24
  21.25     4.45
```

R Session:

```
R version 2.13.1 (2011-07-08)
Copyright (C) 2011 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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  Natural language support but running in an English locale

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Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.41 (5874) i386-apple-darwin9.8.0]

[History restored from /Users/andrejstreibergs/.Rapp.history]

> tt=read.table("M3082DataTruck.txt",header=T)
> attach(tt)
```
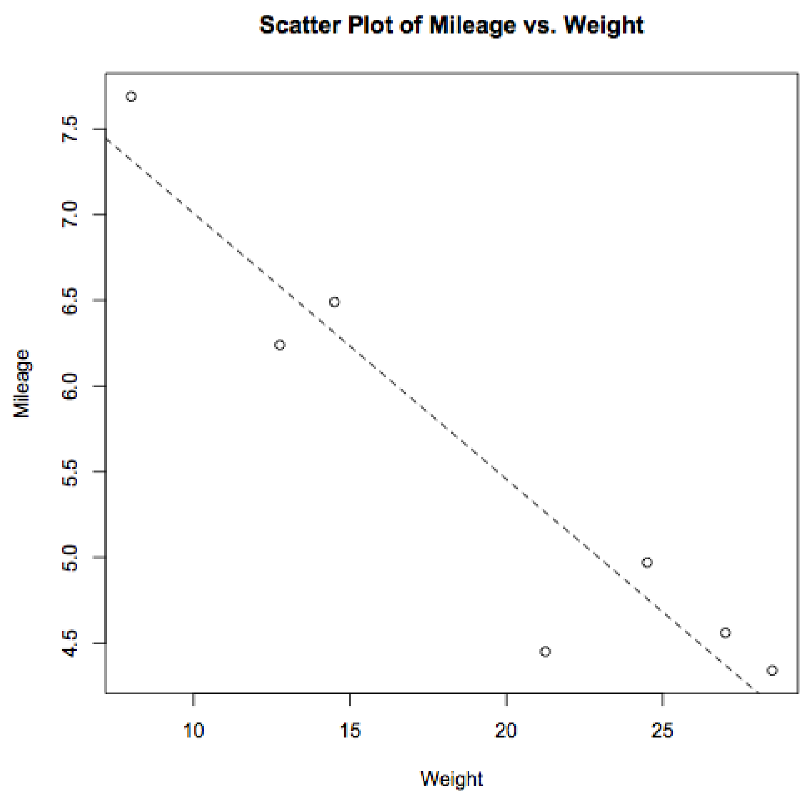

```
> ###############  RUN REGRESSION  ############################
> f1=lm(Mileage~Weight); f1
Call:
lm(formula = Mileage ~ Weight)

Coefficients:
(Intercept)       Weight
     8.5593      -0.1551
> ###############  SCATTERPLOT WITH REGRESSION LINE  ##########
> plot(tt,main="Scatter Plot of Mileage vs. Weight");
> abline(f1,lty=2)
```




```
> ################  COMPUTE ANOVA TABLE 'BY HAND"  #############
> WtBar=mean(Weight); WtBar
[1] 19.5
> MiBar=mean(Mileage); MiBar
[1] 5.534286
> SWt=sum(Weight); SWt
[1] 136.5
> SMi=sum(Mileage); SMi
[1] 38.74
> S2Wt=sum(Weight^2); S2Wt
[1] 3029.875
> S2Mi=sum(Mileage^2); S2Mi
[1] 224.3264
> SWtMi=sum(Weight*Mileage); SWtMi
[1] 698.3225
```

```
> n=length(Weight); n
[1] 7
> Sxx=S2Wt-SWt^2/n; Sxx
[1] 368.125
> Sxy=SWtMi-SWt*SMi/n; Sxy
[1] -57.1075
> b1hat=Sxy/Sxx; b1hat
[1] -0.1551307
> b0hat=MiBar-b1hat*WtBar; b0hat
[1] 8.559335
> SSE = S2Mi -b0hat*SMi -b1hat*SWtMi; SSE
[1] 1.069043
> MSE=SSE/(n-2); MSE
[1] 0.2138087
> SST=S2Mi-MiBar^2/n; SST
[1] 219.9509
> SST=S2Mi-SMi^2/n; SST
[1] 9.928171
> SSR=SST-SSE; SSR
[1] 8.859128
> MSR = MSR
> f=MSR/MSE; f
[1] 41.43484
> PVal=pf(f,1,n-2,lower.tail=F); PVal
[1] 0.001344843
> ########  PRINT THE "ANOVA BY HAND" TABLE  ###############
> at=matrix(c(1,n-2,n-1,SSR,SSE,SST,SSR,MSE,-1,f,-1,-1,PVal,-1,-1),ncol=5)
> colnames(at)=c("      DF","      SS","      MS","       f","   P-Value")
> rownames(at)=c("Weight","Error","Total")
> noquote(formatC(at,width=9,digits=7))

             DF        SS        MS         f     P-Value
Weight        1  8.859128  8.859128  41.43484 0.001344843
Error         5  1.069043 0.2138087        -1        -1
Total         6  9.928171        -1        -1        -1


> ################  COMPARE TO CANNED TABLE  ##############
> anova(f1)
Analysis of Variance Table

Response: Mileage
          Df Sum Sq Mean Sq F value   Pr(>F)
Weight     1 8.8591  8.8591  41.435 0.001345 **
Residuals  5 1.0690  0.2138
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1
>
```

```
> ##########  RESIDUALS, COEFFICIENTS AND R2 BY HAND  ######
>
>  noquote(formatC(matrix(Mileage-b0hat-b1hat*Weight,ncol=7)))
     [,1]   [,2]   [,3]   [,4]   [,5]   [,6]    [,7]
[1,] 0.3717 0.2114 0.1892 0.1801 0.2019 -0.3414 -0.8128

> SSE
[1] 1.069043
> #########  SSE  THE HARD WAY  ############################

> sum((Mileage-b0hat-b1hat*Weight)^2)
[1] 1.069043

> SSR
[1] 8.859128

> ###############  ESTIMATE OF SIGMA SQUARED  ##############

> MSE
[1] 0.2138087

> ##### ESTIMATE OF SIGMA = RESIDUAL STANDARD ERROR  #######

> s = sqrt(MSE); s
[1] 0.4623945

> ###########  COEFFICIENT OF DETERMINATION  ###############
> R2 = SSR/SST; R2
[1] 0.8923222

> ############  ADJUSTED R^2: WEIGHT NUMBER OF COEFF.  #####
> 1-((n-1)*SSE)/((n-1-1)*SST)
[1] 0.8707867

> ##### ESTIMATE STANDARD ERRORS FOR  BETA_0, BETA_1  #######

> s0 = s*sqrt(1/n + WtBar^2/Sxx); s0
[1] 0.5013931

> s1=s/sqrt(Sxx); s1
[1] 0.02409989

> ### T-SCORES, P-VALUES ASSUMING 2-SIDED, NULL HYP. = 0  ##

> t0 = b0hat/s0; t0
[1] 17.07111

> p0 = 2*pt(abs(t0),n-2,lower.tail=F); p0
[1] 1.262291e-05

> t1 = b1hat/s1; t1
[1] -6.43699
```

```
> p1 = 2*pt(abs(t1),n-2,lower.tail=F); p1
[1] 0.001344843

> ###########  COMPARE TO CANNED SUMMARY  ##################
> summary(f1)
Call:
lm(formula = Mileage ~ Weight)

Residuals:
      1       2       3       4       5       6       7
 0.3717  0.2114  0.1892  0.1801  0.2019 -0.3414 -0.8128

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   8.5593     0.5014  17.071 1.26e-05 ***
Weight       -0.1551     0.0241  -6.437  0.00134 **
---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 0.4624 on 5 degrees of freedom
Multiple R-squared: 0.8923,Adjusted R-squared: 0.8708
F-statistic: 41.43 on 1 and 5 DF,  p-value: 0.001345
```