Math $3070 \S 1$.	Pump Price Example: Bootstrap Means are	Name: Example
Treibergs	Normal Even if Sampled from Non-Normal	July 10, 2011

We simulate a distribution Example 5.28 of Devore, *Probability and Statistics for Engineering* and the Sciences, 8th ed., Brooks Cole, 2012. Devore quotes the distribution of prices paid at the gas station given by the article "Data Mining for Fun and Profit," *Statistical Science*, 2000. Apparently, many teenagers pay by even denominations and only partially fill their tanks when they buy gas.

We simulate a sample, x, coming from such a distribution. The histogram of x is given. Then we use nonparametric bootstrapping. We take samples of size n = 16 from x with replacement and compute the sample means. These simulate samples from the actual distribution x is supposed to have come from. We repeat B = 5,000 times and plot a histogram of the means. The resulting bootstrapped sampling distribution of the means is nicely bell-shaped. The QQ-plot lines up nicely indicating that the distribution is close to normal, even though the number is less than the rule of thumb n > 30.

We also run the Shapiro-Wilk test for normality. We reject normality of the means. However, since the number of means is so large, this test is ultra-sensitive to departure of linearity in the QQ-plot.

R Session:

```
R version 2.10.1 (2009-12-14)
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[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]
[Workspace restored from /Users/andrejstreibergs/.RData]
> n <- 16
> B <- 5000
> x <- c((26+11*rt(344,6)),rep(seq(5,60,5),nos),2,3)</pre>
> x <- x[x>0]
> length(x)
[1] 804
> hist(x,breaks=60,freq=F,col=terrain.colors(75))
```

> # M3075PumpPrice3.pdf

```
> y <-replicate(B,mean(sample(x,n,replace=T)))</pre>
>
> summary(y)
 Min. 1st Qu. Median Mean 3rd Qu. Max.
 16.68 24.86 27.14 27.19 29.45 39.92
> sd(y)
[1] 3.370162
> hist(y, freq = F, col=topo.colors(25), xlab = "Mean", breaks = 25,
+ main = paste("Approx Samp Dist of Mean, samp.size=", n, " reps=", B))
> # M3075PumpPrice1.pdf
>
> qqnorm(y)
> qqline(y,col=2)
> # M3075PumpPrice2.pdf
>
> shapiro.test(y)
Shapiro-Wilk normality test
data: y
```

```
W = 0.9989, p-value = 0.001549
```

Histogram of x







Normal Q-Q Plot

Theoretical Quantiles