

In this example we simulate two strategies for playing the Monty Hall Game. This exercise is based on the Math 3070 Lab demonstration for week 7 “The Monty Hall Problem” by Tony Lam. We do a simpler simulation than suggested by Lam.

Monty Hall was the long time host of the TV game show *The Price is Right*. A contestant is presented three curtains. Behind one of the curtains is a prize and behind the others are goats. The contestant is asked to choose one of the curtains and will win whatever is behind it, but in not shown what’s behind the curtain yet. Monty Hall opens one of the other two curtains to reveal a goat. He asks the contestant whether she wishes to change her original choice and pick the other (unopened) curtain. Should she switch?

Here is one analysis of the problem. First consider what happens if her strategy is not to switch. Suppose that the contestant chooses one of the curtains. There is a one in three chance that she has chosen the prize. Now consider what happens if her strategy is to switch. Suppose that the contestant chooses one of the curtains. If the prize is behind this one, then switching will lose the prize. Monty Hall just randomly picks one of the remaining curtains to reveal a goat. On the other hand, if the prize is not behind the first door, and Monty Hall shows the door with the second goat, and the third curtain hides the prize. Switching will select the prize.

Thus without switching, the chances of winning the prize is $1/3$ whereas with switching, the chances of winning the prize is $2/3$.

In our simulation, we have a one in three chance that the prize is behind the first door chosen, which we call Door 1. The contestant gets the prize behind Door 1 if her strategy is not to switch. The contestant gets the prize behind Door 2 or Door 3 (the other doors) if her strategy is to switch.

The solution is not so simple, and really depends on exactly what Monty Hall will do. Here we are assuming that Monty Hall will always show us a goat and that if he has a choice of two goats then he will pick one at random. (See, *e.g.*, Stirzaker, *Elementary Probability*, 2nd. ed., Cambridge, 2003, p. 75.)

R Session:

R version 2.10.1 (2009-12-14)
Copyright (C) 2009 The R Foundation for Statistical Computing
ISBN 3-900051-07-0

R is free software and comes with ABSOLUTELY NO WARRANTY.
You are welcome to redistribute it under certain conditions.
Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]

```

> ##### SIMULATING THE MOINTY HALL PROBLEM #####
> # sample() makes random samples. The possible choices
> 1:3
[1] 1 2 3
> # To take 15 choices from 1:3 with replacement
> sample(1:3,15,replace=TRUE)
[1] 2 2 1 3 2 1 2 2 3 3 3 3 3 1 1
< # Every time sample() is called a different random choice is made:
> rand <- sample(1:3,15,replace=TRUE)
> rand
[1] 3 3 2 2 3 2 3 2 1 1 1 3 2 3 3
> # To see how many times the prize was left behind each door,
> # we tabulate the number of times each choice occurred:
> table(rand)
> x <- table(rand); x
rand
1 2 3
3 5 7
> # If the contestant does not change her her initial pick
> # (say door 1), the number of times the prize is there is the
> # frequency of "1"'s in the sample. To pick off the number of
> # times the prize was behind door 1:
> x[1]
1
3
> # On the other hand, if she chooses another door, she wins if
> # the prize is behind one of the other doors. The total number is
> ChangeWins <- x[2]+x[3]; ChangeWins
2
12
> # The proportion of wins if the choice was changed is
> ChangeWins/15
[1] 0.8

> # Now lets run the simulation n times.
> n <- 2000
> x <- table(sample(1:3,n,replace=TRUE))
> DontChangeWins <- x[1]; Dp <- DontChangeWins/n
> ChangeWins <- x[2]+x[3]; Cp <- ChangeWins/n
>
> # The following code is just to beautify the output. It takes a
> # little fiddling to get it right.

> t1 <- c("\n Monty Hall Problem Simulation\n\n\"Don't Choose Other Door\"\n",
+ " Wins= ")
> t2 <- "\n Number of Trials= "
> t3 <- "\n Observed Winning Proportion= "
> t4 <- "\n Theoretical Winning Proportion= "
> t5 <- "\n\n\"Choose Other Door\"\n Wins= "

```

```
> cat(t1,DontChangeWins,t2,n,t3,Dp,t4,1/3,t5,ChangeWins,t2,n,t3,Cp,t4,2/3,"\n\n")
```

Monty Hall Problem Simulation

"Don't Choose Other Door"

```
                Wins=    685
      Number of Trials= 2000
Observed Winning Proportion=    0.3425
Theoretical Winning Proportion=    0.3333333
```

"Choose Other Door"

```
                Wins=   1315
      Number of Trials= 2000
Observed Winning Proportion=    0.6575
Theoretical Winning Proportion=    0.6666667
```

```
> # Lets bump up the number of trials.
> n <- 20000
> x <- table(sample(1:3,n,replace=TRUE))
> DontChangeWins <- x[1]; Dp <- DontChangeWins/n
> ChangeWins <- x[2]+x[3]; Cp <- ChangeWins/n
> cat(t1,DontChangeWins,t2,n,t3,Dp,t4,1/3,t5,ChangeWins,t2,n,t3,Cp,t4,2/3,"\n\n")
```

Monty Hall Problem Simulation

"Don't Choose Other Door"

```
                Wins=   6715
      Number of Trials= 20000
Observed Winning Proportion=    0.33575
Theoretical Winning Proportion=    0.3333333
```

"Choose Other Door"

```
                Wins=  13285
      Number of Trials= 20000
Observed Winning Proportion=    0.66425
Theoretical Winning Proportion=    0.6666667
```

```

> # Note that another simulation will result in slightly different
> # observed frequencies.
>
> n <- 20000
> x <- table(sample(1:3,n,replace=TRUE))
> DontChangeWins <- x[1]; Dp <- DontChangeWins/n
> ChangeWins <- x[2]+x[3]; Cp <- ChangeWins/n
> cat(t1,DontChangeWins,t2,n,t3,Dp,t4,1/3,t5,ChangeWins,t2,n,t3,Cp,t4,2/3,"\n\n")

```

Monty Hall Problem Simulation

"Don't Choose Other Door"

```

Wins= 6666
Number of Trials= 20000
Observed Winning Proportion= 0.3333
Theoretical Winning Proportion= 0.3333333

```

"Choose Other Door"

```

Wins= 13334
Number of Trials= 20000
Observed Winning Proportion= 0.6667
Theoretical Winning Proportion= 0.6666667

```

> ##### AS A BINOMIAL VARIABLE #####

```

>
> # We are really counting the number of successes in n trials where the
> # probability of success is 1/3. i.e., generating a binomial random variable.
>

```

```

> n <- 200
> DontChangeWins <- rbinom(1,n,1/3); DontChangeWins; DontChangeWins/n
[1] 66
[1] 0.33
> DontChangeWins <- rbinom(1,n,1/3); DontChangeWins; DontChangeWins/n
[1] 62
[1] 0.31
> n <- 20000
> DontChangeWins <- rbinom(1,n,1/3); DontChangeWins; DontChangeWins/n
[1] 6611
[1] 0.33055
> DontChangeWins <- rbinom(1,n,1/3); DontChangeWins; DontChangeWins/n
[1] 6723
[1] 0.33615
> DontChangeWins <- rbinom(1,n,1/3); DontChangeWins; DontChangeWins/n
[1] 6621
[1] 0.33105
> DontChangeWins <- rbinom(1,n,1/3); DontChangeWins; DontChangeWins/n
[1] 6705
[1] 0.33525
> DontChangeWins <- rbinom(1,n,1/3); DontChangeWins; DontChangeWins/n
[1] 6693
[1] 0.33465

```