Homework for Math 3010 §1, Spring 2025

A. Treibergs, Instructor

April 10, 2025

Our text is by Victor J. Katz, *A History of Mathematics, 3rd. ed.*, Pearson India 2019. ISBN: 978-9353433000. Please read the relevant sections in the text as well as any cited references. Assignments are due the following Friday, or on April 19, whichever comes first.

Your written work reflects your professionalism. Homework is to be written legibly on paper. Please copy or paraphrase each question. Make sure your solutions are complete, self contained and written in good technical English. This means that you should write in complete sentences, provide adequate explanation to help the reader understand the structure of your argument, be thorough in the details, state any theorem that you use and proofread your answer. Theorems from the text and notes should be stated, and not merely cited by number. Any sources you use other than the text and notes (such as looking up the answer on line) should be cited.

Homework from Wednesday to Monday will be due Friday. Late homework that is up to one week late will receive half credit. Homework that is more than one week late will receive no credit at all. Homework that is placed in my mailbox in JWB 228 before 4:00 pm Friday afternoon will be considered to be on time. All homework must be handed in by Apr. 30.

> Please hand in on paper problems A1 – A3 on Friday, January 10.

A1. Exercises from Katz's *A History of Mathematics.*

$$28[2, 4, 10]$$

A2.   1. Use the ancient Egyptian procedure to find the area of a circle with diameter 12.

2. What would the area be if the modern value of π were used instead?

3. What percentage error made by the ancient Egyptians?

[Bunt *et. al.* 40[4]]

A3. The Egyptian calculation for the area of an arbitrary quadrilateral is to multiply half the sum of opposite sides by half the sum of the other two sides. Show that the Egyptian procedure for finding the area of a quadrilateral gives the correct result if the quadrilateral is a rectangle and gives too large a number if the figure is a nonrectangular parallelogram or a trapezoid. Is the procedure ever correct for a quadrilateral that is not a rectangle. (Can you prove it?) [Bunt *et. al.* 40[3].]

> Please hand in problems B1 – B3 on Friday, Jan. 17.

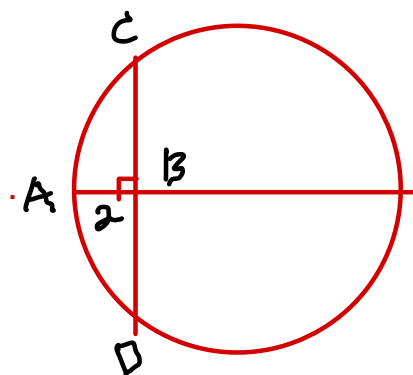**B1.** Exercises from Katz's *A History of Mathematics.*

$$28[17, 18, 20, 21]$$

B2. Compute the sum using Babylonian arithmetic. Convert the summands and your answer to decimals and check that your addition is correct.

$$(5, 51, 12, 49) + (13, 45, 19) = ?$$

B3. From Bunt *et. al.* 63[8].

Solve the following problem that occurs on a Babylonian tablet. Given that the circumference of a circle is 60 and the length of the saggita $\overline{AB}$ is 2, calculate the length of the chord $\overline{CD}$ in the figure.
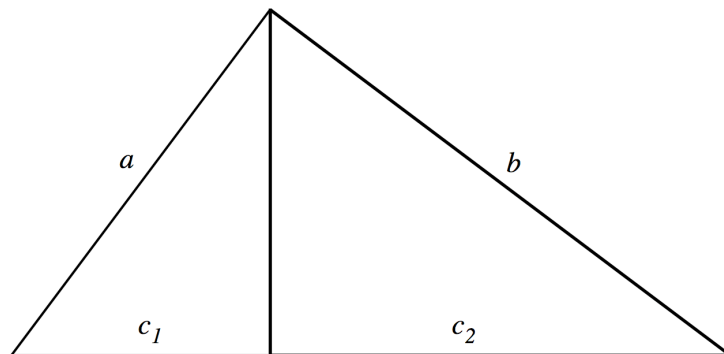

Please hand in problems C1–C3 on Friday, Jan. 24.

C1. Exercises from Katz's *A History of Mathematics*.

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47[7, 9, 10, 12, 21]
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C2. In Book VI of *Elements*, Euclid gives the following argument for the Pythagorean Theorem based on similar triangles. Show that the three triangles in the figure are similar, and hence prove the Pythagorean theorem by equating ratios of corresponding sides. [Stillwell *Mathematics and its History.* p. 10]
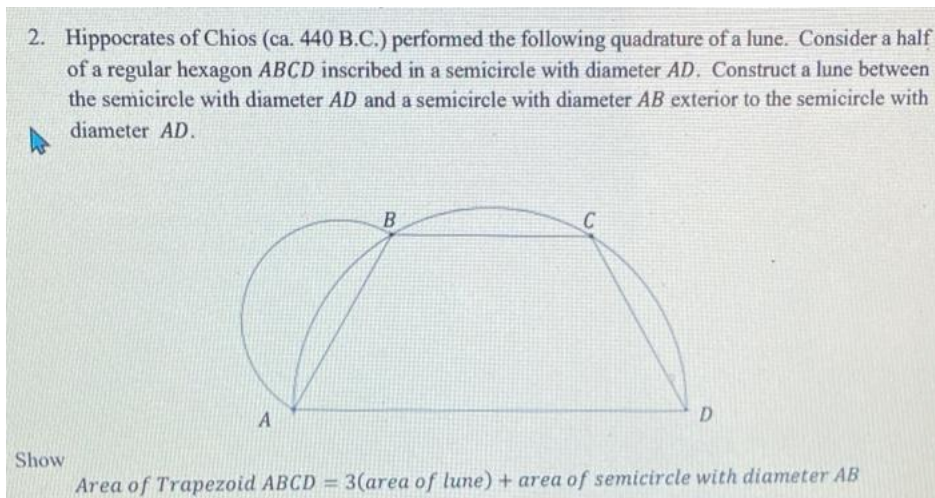

C3. Show that the Golden ratio ϕ is irrational.

D1. Exercises from Katz's *A History of Mathematics.*

90[6, 7 Show area BEFG equals area ABML any way you can, 10, 19]

D2. Find the area of the given lune.

2. Hippocrates of Chios (ca. 440 B.C.) performed the following quadrature of a lune. Consider a half of a regular hexagon *ABCD* inscribed in a semicircle with diameter *AD*. Construct a lune between the semicircle with diameter *AD* and a semicircle with diameter *AB* exterior to the semicircle with diameter *AD*.

Show

Area of Trapezoid ABCD = 3(area of lune) + area of semicircle with diameter AB

D3. On a separate piece of paper, write your Essay on Mathematics of Antiquity proposal. After the proposal is returned to you, please hand your proposal in again when you hand in your essay next week. Be sure to include in your proposal

- Working Title
- Short but specific description of what your essay is about. Don't just say you will discuss what the Greeks thought about π. Better say that you will describe how Archimedes showed that $3\frac{10}{71} < \pi < 3\frac{1}{7}$. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.
- State which style manual you'll follow. You can find a list at the Mariott website http://campusguides.lib.utah.edu/style
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Burton).

E1. Essay on the Mathematics of Antiquity. Write an essay about a specific mathematical discovery/ theorem/ method that occurred before Christ.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.
- There must be some mathematics, and mathematical explanation, in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate.

- You must draw on a bare minimum of three book and journal sources. It is good if you include "primary" sources, quoting directly from the mathematician you're discussing, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the author cited in a Wikipedia article.

- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide. (Parts of these instructions are quoted from Patrikis's assignment 2-19-16.)

- Please attach your approved essay proposal from last week to your paper.

Please hand in problems F1–F3 on Friday, February 14.

F1. Exercises from Katz's *A History of Mathematics.*

128[1, 11, 13]

F2. Show that if $a \mid bc$ and $\gcd(a, b) = 1$ then $a \mid c$.

F3. Here is how Menaechmus constructed the parabola as the locus of points $\mathcal{P} = \{(x, y)\}$. Given points A, O and X on line such that a is the distance AO and x is the distance OX. Let L be a perpendicular line through O. For each x, construct a circle whose diameter is AX. Then y is the distance from O to the intersection point of L with the circle. Show that Menaechmus' consruction yields the parabola. Show also that the parabola passes through the point (a, a).
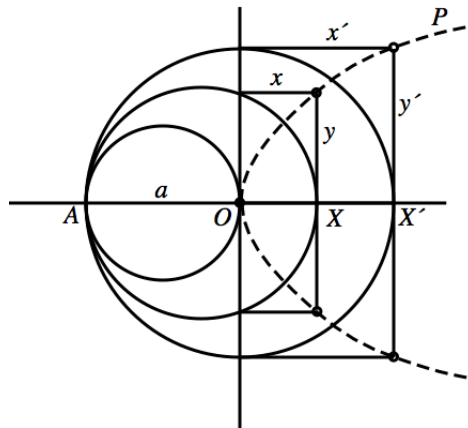
Please hand in problems G1 – G2 on Friday, Feb. 21.

G1. Exercises from Katz's *A History of Mathematics.*

168[1, 4, 22]
226[2, 3, 16 (any method)]

G2. Regiomantus 1436 - 1476: find all numbers that have remainders 3, 11 and 15 when divided by 10, 13 and 17. [Burton, *History of Mathematics,* p. 232.]

Please hand in problems H1 – H2 on Friday, Feb. 28.

H1. Exercises from Katz's *A History of Mathematics.*

$$262[11, 23, 24, 25]$$

H2. Solve the following problem from *Lilavati* of Bhaskara II. "A person gave three *drammas* on the first day, and continued to distribute alms increasing by two [a day]; and he thus bestowed on the priests three hundred and sixty *drammas*: say quickly in how many days?" [Calinger, *Classics of Mathematics,* p. 224.]

Please hand in problems I1 – I3 on Friday, Mar. 7.

I1. Exercises from Katz's *A History of Mathematics.*

$$318[1, 9, 11, 16, 22, 23]$$

I2. Use Brahmagupta's interpolation formula to approximate $\text{Sin}(1000')$ from the early sine table of Aryabhatiya with $R = 3438$.

I3. On a separate piece of paper, write your Essay on Renaissance Mathematics proposal. It should be about a specific mathematical discovery/ theorem/ method that occurred between AD 500 – AD 1850. After the proposal is returned to you, please hand your proposal in again when you hand in your essay in two weeks. Be sure to include in your proposal

- Working Title
- Short but specific description of what your essay is about. Don't just say you will discuss what Omar Khayyam thought about Euclid. Better say you will describe Khayyam's suggestions to replace the parallel postulate which anticipate the modern development of non-Euclidean geometry. Everyone in class should have a different topic.
- State an interesting fact you've discovered about your topic in your preliminary readings.
- State which style manual you'll follow. You can find a list at the Mariott website http://campusguides.lib.utah.edu/style
- Give two internet references. Please include the author and the URL.
- Give two book or journal references specific to your topic (other than Burton). One of your sources MUST BE a primary source, quoting directly from the mathematician being discussed.

> Please hand in your essay J1 on Friday, Mar. 21.

J1. Essay on the Renaissance Mathematics. Write your proposed essay about a specific mathematical discovery/ theorem/ method that occurred between AD 500 – AD 1850.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.

- There must be some mathematics, and mathematical explanation, in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate.

- You must draw on a bare minimum of three book and journal sources. ONE MUST BE a "primary" source, quoting directly from the mathematician you're discussing, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the source cited in a Wikipedia article.

- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide. (Parts of these instructions are quoted from Patrikis's assignment 2-19-16.)

- Please attach your essay proposal from last week to your paper.

> Please hand in problems K1 on Friday, Mar. 28.

K1. Exercises from from Katz's *A History of Mathematics.*

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418[31, 33]
462[19]
501[22, 29, 33]
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> Please hand in problems L1–L3 on Friday, Apr. 4.

L1. Exercises from from Katz's *A History of Mathematics.*

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539[9, 10]
579[9]
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L2. Show that the binomial series gives

$$\frac{1}{\sqrt{1-t^2}} = 1 + \frac{1}{2}t^2 + \frac{1 \cdot 3}{2 \cdot 4}t^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}t^6 + \cdots$$

Then use

$$\sin^{-1} x = \int_0^x \frac{dt}{\sqrt{1-t^2}}$$

to derive Newton's series for $\sin^{-1} x$.

L3. Use Newton's version of Newton's method to approximate the root of $x^2 - 2 = 0$ to an accuracy of eight decimal places.

> Please hand in problems M1 – M2 on Friday, April 11.

M1. Exercises from from Katz's *A History of Mathematics.*

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580[24, 30]
638[24, 27, 33]
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M2. On a separate piece of paper, write your Essay on Modern Mathematics proposal. It should be about a specific mathematical discovery/ theorem/ method that occurred after 1850. After the proposal is returned to you, please hand your proposal in again when you hand in your essay next week. Be sure to include in your proposal

- Working Title

- Short but specific description of what your essay is about. Don't just say you will discuss what the Henri Poincaré thought about the fundamental group. Better to describe not only why Poincaré invented the fundamental group, but say what it is and prove some things about it. Everyone in class should have a different topic.

- State an interesting fact you've discovered about your topic in your preliminary readings.

- State which style manual you'll follow. You can find a list at the Mariott website `http://campusguides.lib.utah.edu/style`

- Give two internet references. Please include the author and the URL.

- Give two book or journal references specific to your topic (other than Katz). One of your sources must be a primary source, quoting directly from the mathematician being discussed.

> Please hand in problems N1 on Wed., Apr. 18.

N1. Essay on the Modern Mathematics. Write an essay about a specific mathematical discovery/ theorem/ method that occurred after 1850.

- The paper should be five pages (in some reasonable font and font size) double-spaced and printed out on paper. It should be in written in good technical English. It should be written for an audience of Math 3010 students.

- There must be some mathematics and mathematical explanation in your paper. Just how you incorporate some mathematical exposition will vary from subject to subject. Include displayed equations and diagrams if appropriate. Your mathematical arguments should be careful and thorough enough to convince the reader that you understand the mathematics.

- You must draw on a bare minimum of three book and journal sources. ONE MUST BE a "primary" source, *quoting directly from the mathematician you're discussing*, or at least from sources closest to them reconstructing the original source. A "Secondary" source is a scholarly interpretation later than the original subject of study in a book or journal. You may use blogs and Wiki articles provided that you give them credit. But also track down the source cited in a Wikipedia article.

- Give credit where it is due: whenever you use another author's ideas, whether appearing in your paper as direct quotation, paraphrase, or simply influence, you must cite them (with a footnote and then include in the bibliography). Formatting these citations and bibliography entries should be unambiguous, according to your chosen style guide.

- Please attach your essay proposal from last week to your paper.

> All outstanding homework and essays are due Apr. 18. No papers will be accepted after Apr. 24.
>
> FINAL EXAM is Thu., Apr. 24 at 1:00 - 3:00 PM in the usual LCB 121.