Math 2270 § 2. **Final Exam Part 1** Name:__________________

Treibergs $\sigma\tau$ December 11, 2020

1.	/15
2.	/15
3.	/15
4.	/15
5.	/15
6.	/15
7.	/15
8.	/15
9.	/15
Total	/105

This is an open book test. You may use your text and notes. Do not use a a calculator or consult with another person or use the internet. Work must be shown to receive credit. Write clearly, justify your answers. PDF answer files must be upload to canvas within the time provided. There are [60] total points for Part 1 of the exam. When you complete Part 1 you may go on to Part 2. **DO ONLY FOUR PROBLEMS FROM PART 1.** If you do more than four problems, only the first four will be graded.

$$A = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 2 & 2 \\ 1 & 4 & 2 & 2 \\ 3 & 7 & 6 & 6 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 2 \\ 4 \\ 8 \\ 14 \end{bmatrix}$$

1. [15] Find the general solution of $A\mathbf{x} = \mathbf{b}$. Work must be shown to receive credit.

2. [15] Find $\det(C)$ where $C = AB$ and

$$A = \begin{bmatrix} 3 & 2 & 0 & 2 \\ 2 & 2 & 2 & 2 \\ 0 & 2 & 0 & 0 \\ 5 & 4 & 3 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 2 & 2 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 3 & 3 & 3 & 4 \\ 0 & 2 & 2 & 2 \end{bmatrix}$$

3. (a) [8] Find all eigenvalues and eigenvectors of A.

   (b) [7] Determine whether A is diagonalizable. If it is, find a matrix P that diagonalizes A and check that P does the job.

$$A = \begin{bmatrix} 7 & 1 & 1 \\ -1 & 5 & -1 \\ -2 & -2 & 4 \end{bmatrix},$$

4. $\mathbf{x}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 1 \\ -1 \\ -2 \\ -4 \end{bmatrix}, \quad \mathbf{x}_4 = \begin{bmatrix} 4 \\ 2 \\ 4 \\ 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 3 \\ 0 \end{bmatrix},$

(a) [7] Let $\mathbb{W} = \operatorname{span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4\}$. Find a basis for $\mathbb{W}$.

(b) [3] Is $\mathbf{b} \in \mathbb{W}$?

(c) [5] Find a basis for $\mathbb{W}^\perp$.

5. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2 \mathbf{b}_3\}$ be a basis for $\mathbf{R}^3$ where

$$\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \ \mathbf{b}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Suppose $T : \mathbf{R}^3 \to \mathbf{R}^3$ is a linear transformation such that

$$T(\mathbf{b}_1) = \mathbf{b}_1 + \mathbf{b}_2, \quad T(\mathbf{b}_2) = \mathbf{b}_2 + \mathbf{b}_3, \quad T(\mathbf{b}_3) = \mathbf{b}_1 - \mathbf{b}_3.$$

(a) [5] Find the matrix of the transformation in the $\mathcal{B}$ basis so that $[T(\mathbf{x})]_{\mathcal{B}} = M[\mathbf{x}]_{\mathcal{B}}$.

(b) [5] Find $[\mathbf{x}]_{\mathcal{B}}$.

(c) [5] Find $T(\mathbf{x})$.

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6. [15] Find the least squares solution to $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

7. Determine whether the following statements are true or false. If true, give a short explanation. If false, find matrices for which the statement fails.

   (a) [5] STATEMENT. If A is an orthogonal $n \times n$ matrix then $\|A\mathbf{x}\| = \|\mathbf{x}\|$ for all $\mathbf{x} \in \mathbf{R}^n$.

TRUE: ◯	FALSE: ◯

   (b) [5] STATEMENT. The set of solutions of a linear equation $\mathcal{S} = \{\mathbf{x} \in \mathbf{R}^n : A\mathbf{x} = \mathbf{b}\}$ is a vector subspace of $\mathbf{R}^n$.

TRUE: ◯	FALSE: ◯

   (c) [5] STATEMENT. Let A and B be $n \times n$ matrices with the same eigenvalues having the same multiplicities. Then there is a matrix C such that $AC = CB$.

TRUE: ◯	FALSE: ◯

8. Let $\mathcal{B} = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ be a basis for $\mathbf{R}^3$ where

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \ \mathbf{x}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \ \mathbf{x}_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}.$$

(a) [12] Apply the Gram Schmidt algorithm to $\mathcal{B}$ to find an orthonormal basis.

(b) [3] Let $\mathcal{S} = \operatorname{span}(\mathbf{x}_1, \mathbf{x}_2)$. Find $\operatorname{proj}_{\mathcal{S}} \mathbf{v}$.

9. [15] Find a matrix that orthogonally diagonalizes $A = \begin{bmatrix} 3 & 4 \\ 4 & 9 \end{bmatrix}$ and check that your matrix does the job.