Name: ________________________________

This test has 6 questions, for a total of 100 points.
Point values are indicated for each problem.
Work neatly and carefully. Show all of your work unless otherwise instructed.
No calculator, no texts, no notes, no cheating.

Question	Points	Score
1	15	
2	25	
3	12	
4	16	
5	12	
6	20	
Total:	100	

1. (15 pts) **True/False:** Suppose elimination turns the matrix A into the following matrix in reduced row echelon form:

$$R = \begin{bmatrix} 0 & 1 & -7 & 0 & 0 & 8 & -1 \\ 0 & 0 & 0 & 1 & 0 & 8 & 2 \\ 0 & 0 & 0 & 0 & 1 & 7 & -5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Clearly label each statement below as **True** or **False**:

(a) _______________: The dimension of the nullspace of A equals the dimension of the nullspace of R.

(b) _______________: The column space of A is equal to the column space of R.

(c) _______________: The nullspace of A is a subspace of $\mathbb{R}^4$.

(d) _______________: The dimension of $C(A^T)$ is 3.

(e) _______________: The dimensions of the four fundamental subspaces of A add up to 11.

2. **Short Answer:** For each question below you need give justification only if requested.

   (a) (5 pts) Define what it means to say that vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ in $\mathbb{R}^m$ are *linearly independent.*

   (b) (5 pts) The line $y = C + Dx$, where C and D are given by

$$\begin{bmatrix} C \\ D \end{bmatrix} = \left(\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 4 \\ 1 & 4 \\ 1 & 7 \end{bmatrix} \right)^{-1} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 4 & 4 & 7 \end{bmatrix} \begin{bmatrix} -3 \\ -2 \\ 5 \\ 3 \\ 6 \end{bmatrix},$$

   is the line of best fit through what collection of points in the plane?

   (c) (5 pts) Give an example of a 3×3 matrix A with $\det(A) = 0$, but no entry of A is the number 0. Briefly justify your answer.

Continue on next page. . .

(d) (5 pts) Use Cramer's Rule to find x_1 in the solution to

$$\begin{bmatrix} 6 & -3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}.$$

(e) (5 pts) Define what it means for a matrix Q to be *orthogonal*, and give an example of an orthogonal matrix other than the identity.

3. (12 pts) Find a basis for the orthogonal complement in $\mathbb{R}^4$ to the subspace V spanned by the vectors

$$\mathbf{v}_1 = (1, -1, 2, 0) \qquad \text{and} \qquad \mathbf{v}_2 = (0, 0, 1, -1).$$

4. (16 pts) Use the process of Gram–Schmidt to find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{x}_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \qquad \text{and} \qquad \mathbf{x}_3 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ -1 \end{bmatrix}.$$

5. (12 pts) Find the volume of the parallelepiped in $\mathbb{R}^3$ with one corner at the origin and sides formed by the vectors $(1, -2, -1)$, $(2, 0, 2)$, and $(3, 1, 2)$. (Hint: This is a determinant question.)

6. (20 pts) Diagonalize the matrix

$$A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}.$$

That is, find matrices S and Λ so that $A = S\Lambda S^{-1}$.