Name: ______________________________

This test has 5 questions, for a total of 100 points.
Point values are indicated for each problem.
Work neatly and carefully. Show all of your work unless otherwise instructed.
No calculator, no texts, no notes, no cheating.

Question	Points	Score
1	35	
2	15	
3	15	
4	20	
5	15	
Total:	100	

1. (35 pts) This problem asks you to perform our elimination algorithm on the matrix

$$A = \begin{bmatrix} 2 & -4 & 0 \\ -3 & 7 & -1 \\ -2 & 5 & \frac{1}{2} \end{bmatrix}$$

   (a) Perform Gaussian elimination, following the algorithm presented in class, to turn A into a matrix U in echelon form. **Show your work.** For each step, write clearly in words what row operation you are performing, and write down the matrix whose multiplication accomplishes the row operation.

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(b) Write the LU decomposition of A. That is, find a matrix L that is lower triangular with 1's on the main diagonal so that $A = LU$.

(c) Rewrite the matrix U from part (a). How many pivots does U have? Label the pivots of the matrix clearly.

(d) Is the matrix A invertible? Justify your answer. If it is invertible, explain how you would find its inverse.

2. (15 pts) Consider two vectors $\mathbf{v} = (1, -3, 2, 0, 1)$ and $\mathbf{w} = (-2, 0, 2, 2, 2)$ in $\mathbb{R}^5$.

   (a) Compute $\|\mathbf{v}\|$, $\|\mathbf{w}\|$, and $\mathbf{v} \cdot \mathbf{w}$.

   (b) Let θ denote the angle between $\mathbf{v}$ and $\mathbf{w}$. Write the formula for $\cos(\theta)$. Is θ an acute, obtuse, or right angle?

3. (15 pts) Consider the following matrices:

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}.$$

Calculate each of the following, if it makes sense.

(a) BA

(b) $A^T B$

(c) $A \begin{bmatrix} 1 \\ -5 \end{bmatrix}$

4. (20 pts) This problem asks you to find the complete solution to the system of equations

$$\begin{aligned} 2x_2 + 4x_3 &= 12 \\ x_1 - 2x_2 - 3x_3 &= -9 \end{aligned} \tag{1}$$

(a) Perform Gauss Jordan elimination to turn the matrix

$$A = \begin{bmatrix} 0 & 2 & 4 & 12 \\ 1 & -2 & -3 & -9 \end{bmatrix}$$

into a matrix R in reduced row echelon form.

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(b) Consider the system

$$\begin{aligned} 2x_2 + 4x_3 &= 0 \\ x_1 - 2x_2 - 3x_3 &= 0 \end{aligned} \tag{2}$$

What are the free variables in system 2? Find all 'special solutions' to system 2. (Hint: Use your answer to part (a).)

(c) Find the complete solution to system 1 by first finding a particular solution, then using your answer to part b.

5. (15 pts) Fix a vector $\mathbf{v} = (3, -1, 3, -2)$. Which of the following are subspaces of $\mathbb{R}^4$? **Fully justify your answer.**

   (a) The set W of all vectors $\mathbf{w} = (w_1, w_2, w_3, w_4)$ so that $\mathbf{v} \cdot \mathbf{w} = 0$.

   (b) The set X of all vectors $\mathbf{x} = (x_1, x_2, x_3, x_4)$ so that $\mathbf{v} \cdot \mathbf{x} = 1$.