Math 2270 § 3. **First Midterm Exam** Name: Sample

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The following questions are typical of what you might expect on the midterm exam. The answer of some questions may be useful to simplify the answer others. A midterm exam might consist of a subset of the problems, *e.g.* $\{(2),(4),(6),(8),(10)\}$.

$$A=\begin{pmatrix}1&2&3\\2&-1&3\\1&-1&2\end{pmatrix},\ B=\begin{pmatrix}1&1&2&2\\1&2&3&1\\1&4&2&2\\1&-3&10&-6\end{pmatrix},\ c=\begin{pmatrix}1\\4\\7\end{pmatrix},\ d=\begin{pmatrix}3\\4\\8\\-9\end{pmatrix},\ v=\begin{pmatrix}1\\2\\3\\4\end{pmatrix}$$

1. Solve $Ax=c$ using Gauss-Jordan elimination.
2. Find A^{-1} using Gauss-Jordan elimination.
3. Solve $Ax=c$ using the inverse matrix.
4. Find all solutions if any of $Bx=d$.
5. Find all solutions if any of $Bx=v$.
6. Find $\det A$.
7. Find $\det\begin{pmatrix}1&1&1&1&1\\1&2&2&2&2\\1&2&3&3&3\\1&2&3&4&4\\1&2&3&4&5\end{pmatrix}$.
8. Solve $Ax=[0,5,0]^T$ using Cramer's rule.
9. Find A^{-1} using cofactors.
10. Let M be an $m\times n$ matrix. Show that if $Mx=w$ has two solutions then it has infinitely many.
11. Show that the inverse of the transpose of a square matrix Q is the transpose of the inverse:
$$(Q^T)^{-1}=(Q^{-1})^T.$$
12. Show that if a 4×3 matrix M is row equivalent to $\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&0\end{pmatrix}$ then the system $Mx=k$ either has no solution or has exactly one solution. What conditions on k guarantee that there is a solution at all?