This is a **closed book** exam. No books, papers or electronic notes. Calculators are permitted but are unnecessary. Be sure to give complete explanations and provide all intermediate steps. Answers must be justified to receive credit. There are [100] total points.

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1.[20] Find the equation of the plane which contains the point (5,1,-6) and the line

$$\frac{x-2}{5} = \frac{y+3}{7}; \qquad z = 4.$$

2.[20] Let $T:\mathbf{R}^2 o \mathbf{R}^3$ be a linear transformation which takes

$$T\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}1\\0\\-1\end{pmatrix}, \qquad T\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}-1\\0\\-1\end{pmatrix}.$$

Find the matrix of the transformation. What is $T \begin{pmatrix} 4 \\ 3 \end{pmatrix}$?

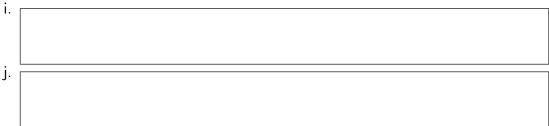
3.[20] Does the set ${\mathcal S}$ span ${\mathbf R}^3$? Why?

$$S = \left\{ \begin{pmatrix} 1\\1\\-1 \end{pmatrix}, \begin{pmatrix} 1\\2\\1 \end{pmatrix}, \begin{pmatrix} 1\\4\\5 \end{pmatrix}, \begin{pmatrix} 5\\3\\-9 \end{pmatrix} \right\}$$

4.[20] Let \mathcal{P}_2 denote the three dimensional vector space consisting of all polynomials of degree two or less. Is the set \mathcal{U} linearly independent? Why? Is the set \mathcal{U} a basis for \mathcal{P}_2 ? Why?

$$\mathcal{U} = \left\{ 2 + t + t^2, \ 1 + 2t + t^2, \ 1 + t + 2t^2 \right\}$$

- 5a.[5] Let $\mathcal{V}=\mathcal{C}(\mathbf{R})$ be the set of all <u>continuous</u> real valued functions of a real variable. Define vector addition to be the standard addition of functions and scalar multiplication to be the standard multiplication of functions. (For $f,g\in\mathcal{V}$ and $c\in\mathbf{R}$, $f\oplus g$ is the function defined by $(f\oplus g)(t)=f(t)+g(t)$ and $c\odot f$ is the function defined by $(c\odot f)(t)=cf(t)$.) Then $(\mathcal{V},\oplus,\odot)$ is a real vector space. Fill in the missing axioms of a real vector space:
 - a. \mathcal{V} is closed under the operation \oplus .
 - b. $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$ for all \mathbf{u}, \mathbf{v} in \mathcal{V} .
 - c. $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$ for all $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in \mathcal{V} .
 - d. There is an element $O \in \mathcal{V}$ such that $u \oplus O = O \oplus u = u$ for all u in \mathcal{V} .
 - e. \mathcal{V} is closed under the operation \odot .
 - f. $(c+d) \odot \mathbf{u} = (c \odot \mathbf{u}) \oplus (d \odot \mathbf{u})$ for all c, d in \mathbf{R} and \mathbf{u} in \mathcal{V} .
 - g. $c \odot (\mathbf{u} \oplus \mathbf{v}) = (c \odot \mathbf{u}) \oplus (c \odot \mathbf{v})$ for all c in \mathbf{R} and \mathbf{u}, \mathbf{v} in \mathcal{V} .
 - h. $1 \odot \mathbf{u} = \mathbf{u}$ for all \mathbf{u} in \mathcal{V} .



b.[5] Prove that $(\mathcal{V}, \oplus, \odot)$ satisfies axioms a. and g.

c.[5] Prove that $(\mathcal{V},\oplus,\odot)$ satisfies axioms a. and g.

d.[5] Let $\mathcal{W}=\{(x,y)\in\mathbf{R}^2:xy\geq0\}$ be the subset of the usual vector space $(\mathbf{R}^2,+,\cdot)$ consisting of the union of the closed first and third quadrants of the plane. Determine whether $(\mathcal{W},+,\cdot)$ is a vector subspace of \mathbf{R}^2 . Why?