Math 2270 § 3.	**Second Midterm Exam**	Name:____________________
Treibergs	October 21, 1998	Student No.:

This is a **closed book** exam. No books, papers or electronic notes. Calculators are permitted but are unnecessary. Be sure to give complete explanations and provide all intermediate steps. Answers must be justified to receive credit. There are [100] total points.

1.______	/20
2.______	/20
3.______	/20
4.______	/20
5.______	/20
Total______	/100

1.[20] Find the equation of the plane which contains the point $(5, 1, -6)$ and the line

$$\frac{x-2}{5} = \frac{y+3}{7}; \qquad z = 4.$$

2.[20] Let $T : \mathbf{R}^2 \to \mathbf{R}^3$ be a linear transformation which takes

$$T\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\-1\end{pmatrix}, \qquad T\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}-1\\0\\-1\end{pmatrix}.$$

Find the matrix of the transformation. What is $T\begin{pmatrix}4\\3\end{pmatrix}$?

3.[20] Does the set $\mathcal{S}$ span $\mathbf{R}^3$? Why?

$$\mathcal{S} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \\ -9 \end{pmatrix} \right\}$$

4.[20] Let $\mathcal{P}_2$ denote the three dimensional vector space consisting of all polynomials of degree two or less. Is the set $\mathcal{U}$ linearly independent? Why? Is the set $\mathcal{U}$ a basis for $\mathcal{P}_2$? Why?

$$\mathcal{U} = \left\{2 + t + t^2,\ 1 + 2t + t^2,\ 1 + t + 2t^2\right\}$$

5a.[5] Let $\mathcal{V} = \mathcal{C}(\mathbf{R})$ be the set of all *continuous* real valued functions of a real variable. Define vector addition to be the standard addition of functions and scalar multiplication to be the standard multiplication of functions. (For $f, g \in \mathcal{V}$ and $c \in \mathbf{R}$, $f \oplus g$ is the function defined by $(f \oplus g)(t) = f(t) + g(t)$ and $c \odot f$ is the function defined by $(c \odot f)(t) = cf(t)$.) Then $(\mathcal{V}, \oplus, \odot)$ is a real vector space. Fill in the missing axioms of a real vector space:

a. $\mathcal{V}$ is closed under the operation $\oplus$.
b. $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$ for all $\mathbf{u}, \mathbf{v}$ in $\mathcal{V}$.
c. $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$ for all $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in $\mathcal{V}$.
d. There is an element $\mathbf{O} \in \mathcal{V}$ such that $\mathbf{u} \oplus \mathbf{O} = \mathbf{O} \oplus \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in $\mathcal{V}$.
e. $\mathcal{V}$ is closed under the operation $\odot$.
f. $(c + d) \odot \mathbf{u} = (c \odot \mathbf{u}) \oplus (d \odot \mathbf{u})$ for all c, d in $\mathbf{R}$ and $\mathbf{u}$ in $\mathcal{V}$.
g. $c \odot (\mathbf{u} \oplus \mathbf{v}) = (c \odot \mathbf{u}) \oplus (c \odot \mathbf{v})$ for all c in $\mathbf{R}$ and $\mathbf{u}, \mathbf{v}$ in $\mathcal{V}$.
h. $1 \odot \mathbf{u} = \mathbf{u}$ for all $\mathbf{u}$ in $\mathcal{V}$.
i.
j.

b.[5] Prove that $(\mathcal{V}, \oplus, \odot)$ satisfies axioms a. and g.

c.[5] Prove that $(\mathcal{V}, \oplus, \odot)$ satisfies axioms a. and g.

d.[5] Let $\mathcal{W} = \{(x, y) \in \mathbf{R}^2 : xy \ge 0\}$ be the subset of the usual vector space $(\mathbf{R}^2, +, \cdot)$ consisting of the union of the closed first and third quadrants of the plane. Determine whether $(\mathcal{W}, +, \cdot)$ is a vector subspace of $\mathbf{R}^2$. Why?