Math 2270 § 3. **First Midterm Exam** Name:______________________

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1. [30] Find all solutions of the system of linear equations.

$$\begin{aligned} 2x_1 + x_2 + 3x_3 + 3x_4 &= 6 \\ x_1 + 2x_2 + 2x_3 + 1x_4 &= 2 \\ 3x_1 - 6x_2 + 2x_3 - 2x_4 &= -1 \end{aligned}$$

2. (a) [15] Compute A^{-1} using Gauss-Jordan elimination, where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 0 & 5 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) [15] Solve $Ax = b$ for x using Cramer's rule. (Using another method will score zero.)

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad b = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$$

3. (a) [4] Let $A = [a_{ij}]$ denote an $n \times n$ matrix. Write the *definition* of $\det(A)$.

(b) [11] Suppose that A is an $n \times n$ matrix whose first column consists of zeros. Using your definition in part (a.), show that $\det(A) = 0$.

$$A = \begin{pmatrix} 0 & a_{1,2} & \cdots & a_{1,n} \\ 0 & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{n,2} & \cdots & a_{n,n} \end{pmatrix}$$

4. [10] Find

$$\begin{vmatrix} 2 & 0 & 3 & 0 & 5 \\ 0 & 2 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 \end{vmatrix}.$$

5. (a) [5] Let $X = [x_1, x_2, \ldots, x_n]$ be a matrix with 1 row and n columns. Let X^T denote the transpose. Find $X^T X$. [Hint: be careful.]

(b) [10] Let A be an $n \times n$ matrix. Suppose that for some vector b the system $Ax = b$ has no solution. Show that $Ax = 0$ has infinitely many solutions. (One of our theorems tell us that a square matrix for which the inhomogeneous equation can't be solved is singular, therefore the homogeneous equation has infinitely many solutions. You are to explain or prove why this is the case.) [Hint: row operations.]