

This is a closed book test. No other books, papers, calculators, tablets, laptops, phones or other messaging devices are permitted. Give complete solutions. Be clear about your logic and definitions and justify any theorems that you use.

Total \_\_\_\_\_/25

1. Use integration by parts to compute the following integrals.

$$(a) [13] \int_0^1 x e^{2x} dx. = \int_0^1 u dv = [uv]_0^1 - \int_0^1 v du$$

$$= \left[ \frac{x}{2} e^{2x} \right]_0^1 - \int_0^1 \frac{e^{2x}}{2} dx$$

let  $u = x$   $dv = e^{2x} dx$   
 $du = dx$   $v = \frac{e^{2x}}{2}$

$$= \left[ \frac{x}{2} e^{2x} - \frac{e^{2x}}{4} \right]_0^1$$

$$= \frac{1 \cdot e^{2 \cdot 1}}{2} - \frac{e^{2 \cdot 1}}{4} - \frac{0 \cdot e^{2 \cdot 0}}{2} + \frac{e^{2 \cdot 0}}{4}$$

$$= \boxed{\frac{e^2}{4} + \frac{1}{4}}$$

$$(b) [12] \int x \ln(x) dx. = \int u dv = uv - \int v du$$

let  $u = \ln(x)$   $du = \frac{1}{x} dx$   
 $dv = x dx$   $v = \frac{1}{2} x^2$

$$= \frac{1}{2} x^2 \ln(x) - \int \frac{1}{2} x^2 \cdot \frac{1}{x} dx$$

$$= \frac{1}{2} x^2 \ln(x) - \frac{1}{2} \int x dx$$

$$= \frac{1}{2} x^2 \ln(x) - \frac{1}{2} \left( \frac{1}{2} x^2 \right) + C$$

$$= \boxed{\frac{1}{2} x^2 \ln(x) - \frac{1}{4} x^2 + C}$$