Math 1310 § 4. **Third Quiz** Name:____________________

Treibergs March 15, 2023.

This is a closed book test. No other books, papers, calculators, tablets, laptops, phones or other messaging devices are permitted. Give complete solutions. Be clear about your logic and definitions and justify any theorems that you use.

Total_____/25

1. Find the following limits. If you use l'Hospital's Rule, explain why the conditions for its use are satisfied.

(a) [8] $\lim_{x\to 0+} \dfrac{\sin(2x)}{1-\sec x}$. $\overset{l'H}{=} \lim_{x\to 0+} \dfrac{\cos(2x)\cdot 2}{0-\sec x \tan x} = \dfrac{\cos(0)\cdot 2}{0-\sec 0 \tan 0} = \dfrac{2}{-0} = \boxed{-\infty}$

positive

negative for $x>0$.

$\sin(2x) \to \sin(2\cdot 0) = 0$

and

$1-\sec(x) \to 1-\sec(0) = 1-1 = 0$

as $x \to 0+$. Hence

limit is "$\frac{0}{0}$" type and l'Hospital's Rule is valid.

(b) [8] $\lim_{x\to \frac{\pi}{2}-} \dfrac{\cos x}{\csc x}$. $= \dfrac{\cos(\frac{\pi}{2})}{\csc(\frac{\pi}{2})} = \dfrac{0}{1} = \boxed{0}$

$\cos x \to 0$ and

$\csc x \to 1$ as $x \to \frac{\pi}{2}-$

not "$\frac{0}{0}$" form. l'Hospital's rule is inappropriate.

(c) [9] $\lim_{x\to 1+} \left\{ \dfrac{1}{x-1} - \dfrac{1}{\ln x} \right\}$. $= \lim_{x\to 1+} \dfrac{\ln(x)-(x-1)}{(x-1)\ln x} \overset{l'H}{=} \lim_{x\to 1+} \dfrac{\frac{1}{x}-1}{\ln x + \frac{x-1}{x}}$

$\frac{1}{x-1} \to +\infty$ and

$\frac{1}{\ln x} \to +\infty$ as $x \to 1+$

"$\infty - \infty$" type

$\ln x - (x-1) \to 0-0=0$

and

$(x-1)\ln x \to 0\cdot 0 = 0$

as $x \to 1+$

"$\frac{0}{0}$" type

ok to apply l'H.

$\frac{1}{x} - 1 \to \frac{1}{1} - 1 = 0$

and

$\ln x + \frac{x-1}{x} \to 0 + \frac{1-1}{1} = 0$

as $x \to 1+$

"$\frac{0}{0}$" type

$$\overset{l'H}{=} \lim_{x\to 1+} \frac{-\frac{1}{x^2}}{\frac{1}{x} + \frac{1\cdot x - (x-1)\cdot 1}{x^2}} = \lim_{x\to 1+} \frac{-\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}} = \frac{-\frac{1}{1}}{\frac{1}{1}+\frac{1}{1^2}} = \boxed{-\frac{1}{2}}$$