

This is a closed book test. No other books, papers, calculators, tablets, laptops, phones or other messaging devices are permitted. Give complete solutions. Be clear about your logic and definitions and justify any theorems that you use.

Total _____/25

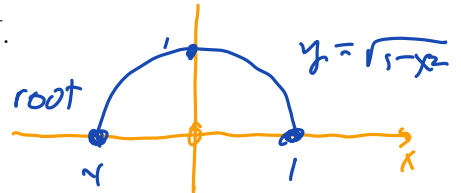
Answer the following questions about the function $f(x) = \sqrt{1-x^2}$.

1. [5] What is the domain of f ? What is the range of f ? *Square root*

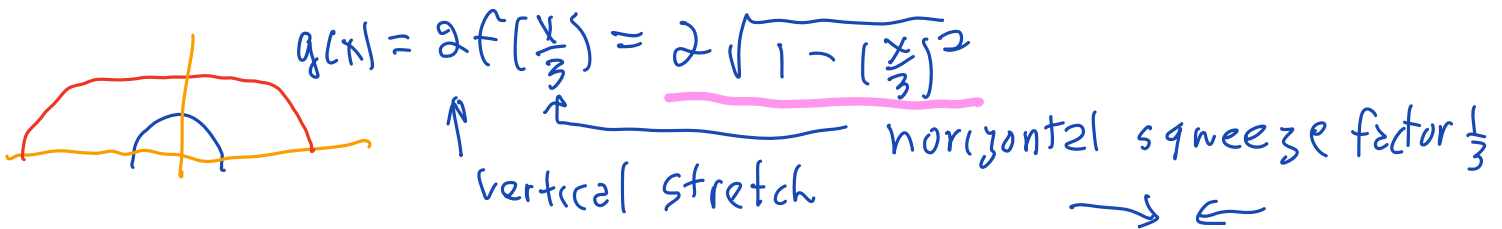
needs $1-x^2 \geq 0$ or $1 \geq x^2$, so $-1 \leq x \leq 1$.

domain is $[-1, 1]$.

range $0 \leq f(x) \leq 1$ so range = $[0, 1]$.



2. [5] Let $g(x)$ be a stretched out version that is twice as high and three times as wide as f . Write a formula for $g(x)$ in terms of $f(x)$.



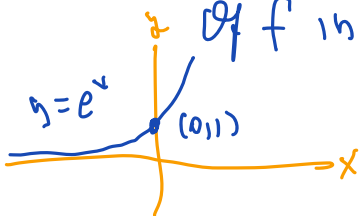
3. [5] Find $f \circ f(x)$.

$$\begin{aligned} f \circ f(x) &= f(f(x)) = f(\sqrt{1-x^2}) = \sqrt{1-(\sqrt{1-x^2})^2} = \sqrt{1-(1-x^2)} \\ &= \sqrt{x^2} = |x|. \quad (\text{Not } x \text{ since } \sqrt{u} \text{ is positive.}) \end{aligned}$$

4. [5] Is f odd? Why? Even? Why?

$$\begin{aligned} f(-x) &= \sqrt{1-(-x)^2} = \sqrt{1-x^2} = f(x) \quad \text{so } f \text{ is } \underline{\text{even}}. \\ -f(x) &= -\sqrt{1-x^2} \quad \text{so } f(-x) \neq -f(x) \text{ so } f \text{ is } \underline{\text{not odd}}. \end{aligned}$$

5. [5] What is the domain of $h(x) = f(e^x)$? e^x has to be within the domain of f in order for $f(e^x)$ to make sense, so $-1 \leq e^x \leq 1$.



Hence $x \leq 0$ so domain of h is $(-\infty, 0]$