Midterm Exam 3 Practice Problems

1. An electrical circuit contains two variable-resistance resistors in series, with resistances $R_1\Omega$ and $R_2\Omega$. It is known that the total resistance R may be expressed as

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}. \tag{1}$$

The first resistance R_1 is equal to $\frac{4}{3}\Omega$, and is decreasing at a rate of $4\Omega/hr$. The second resistance R_2 is equal to 4Ω and is increasing at a rate of $8\Omega/hr$.

(i) What is the total resistance R?

(ii) What is the rate of change of R in Ω/hr?

2. Let $f(x) = \frac{2}{3}x^3 - \frac{3}{2}x^2 - 2x + 10$.

(i) Find the extreme points of f. Identify which of these are local maxima and which are local minima, and explain your answer.

(ii) Find the intervals over which f is increasing, and the intervals where f is decreasing.

(iii) Find any inflection points, and intervals where f is concave upwards, and intervals where f is concave downwards.

(iv) Sketch the curve.

3. Let $f(x) = x^{\frac{1}{3}}(x-1)^2$. Identify the critical numbers of f.

4. Let $f(x) = \cos(x) + 1 - x$. Find the absolute maximum and absolute minimum values of f over the interval $[-\pi, 2\pi]$.

5. Identify the indeterminate form and compute the limit.

$$\lim_{x\to 3} \frac{x-3}{\ln(4-x)}$$

6. Identify the indeterminate form and compute the limit.

$$\lim_{x\to\infty} \frac{e^x}{\ln(x)}$$

7. Identify the indeterminate form and compute the limit.

$$\lim_{x\to \frac{\pi}{2}^-} (x - \frac{\pi}{2})\tan(x)$$

8. Identify the indeterminate form and compute the limit.

$$\lim_{x\to -2} \left(1 + \frac{2}{x}\right)^{x+2}$$

9. Identify the indeterminate form and compute the limit.

$$\lim_{x\to 0^+} (1 + \sin(2x))^{\frac{1}{x}}$$

10. Identify the indeterminate form and compute the limit.

$$\lim_{x\to\infty}\left(\frac{x^2+4}{x}\right)^{\frac{1}{\ln(x)}}$$

11. Identify the indeterminate form and compute the limit.

$$\lim_{x\to 0^+}\left(\frac{1}{x^2}-\frac{1}{\tan(x)}\right)$$

12. Identify the indeterminate form and compute the limit.

$$\lim_{x\to\infty}\left(\ln(x+1)-\ln(\ln(x))\right)$$

13. Identify the indeterminate form and compute the limit.

$$\lim_{x\to 0}\frac{\sin(x)^2}{x\ln(1-x)}$$

14. A canned soup firm wants to redesign their 1 l size cans, so that they minimize the materials used. They want to give it the shape of a cylinder. Find the dimensions of the new can.
    (a) Assign symbols to the relative quantities. Which quantity is to me optimized? Which quantities are variable?
    (b) Find the objective function, that is a function for the to be optimized quantity.
    (c) Find a constraint equation, that is an equation relating all the variable quantities, such that the objective function only depends on one variable quantity.
    (d) Optimize the objective function.
    (e) Relate your solution to the original question.
15. Find two numbers a, b such that $a^2+b^2=25$ and the sum is maximal.
16. Find an algorithm to approximate $\sqrt[4]{12}$.
    (a) Find a polynomial function with integer coefficients, for which $\sqrt[4]{12}$ is a root.
    (b) Give the iteration formula and an initial approximation.
17. Explain, why Newton's method does not work for finding a root of $f(x)=2x^3-3x^2-12x+18$, if the initial approximation is $x_1=2$.
18. Use Newton's method to approximate the positive root of $f(x)=x^2-7$.
    (a) Give the iteration formula.
    (b) Perform two iterations using the initial approximation $x_1=1$.
19. Verify, that $(x^2-2x+2)e^x$ is an antiderivative of x^2e^x.
20. Find an antiderivative of the following functions.
    (a) $f(x)=3x^4+7$
    (b) $f(x)=\sin(x)$
    (c) $f(x)=\dfrac{1}{x}+e^x$
21. Find the most general antiderivative of the following functions.

(a) $f(x) = 3x^2 - e^x$

(b) $f(x) = \cos(x) - \dfrac{1}{x^2}$

22. Consider the function $f(x) = 2\sin(x)\cos(x)$.

    (a) Verify, that $F_1(x) = -\cos(x)^2$ is an antiderivative of $f(x)$.

    (b) Verify, that $F_2(x) = \sin(x)^2$ is an antiderivative of $f(x)$.

    (c) How is it possible, that $F_1(x)$ and $F_2(x)$ are both antiderivatives of $f(x)$.

23. Consider the function

$$f(x) = \begin{cases} x, & x \leq 1 \\ x^2, & x > 1 \end{cases}.$$

    (a) Verify the function is continuous everywhere, particularly at $x = 1$.

    (b) Find an antiderivative $F^-(x)$ of $f(x)$ at points $x \leq 1$.

    (c) Find an antiderivative $F^+(x)$ of $f(x)$ at points $x > 1$.

    (d) Compute $\int_0^1 f(x)dx$.

    (e) Compute $\int_1^2 f(x)dx$.

    (f) Explain why $\int_0^2 f(x)dx$ exists, but is NOT equal to $F^+(2) - F^-(0)$.

24. Using summation "sigma" notation, write down the sum S of sequential integers 3 through 12 in the following ways:

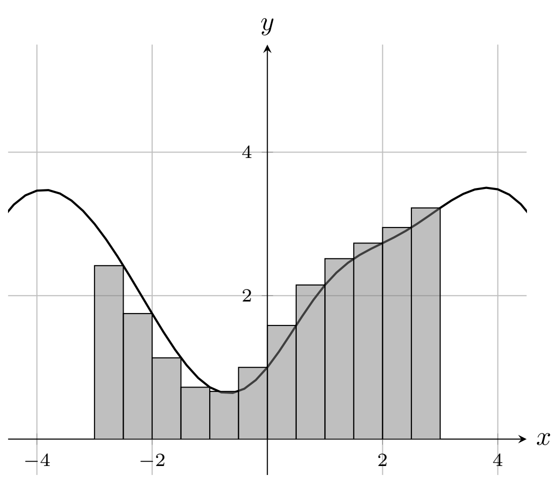
    (a) Using an index that starts at $n = 3$ and ends at $n = 12$. Specify exactly your indexed set A.

    (b) Using an index that starts at $n = 0$ and ends at $n = 9$.Specify exactly your indexed set A.

25. Using summation "sigma" notation, write down the sum S of sequential multiples of 5 starting at 25 through 50. Write out the index set $A = \{a_n\}$ and the explicit sum "$\sum$" indicating starting and ending indexes n. There are many correct ways to index and sum.

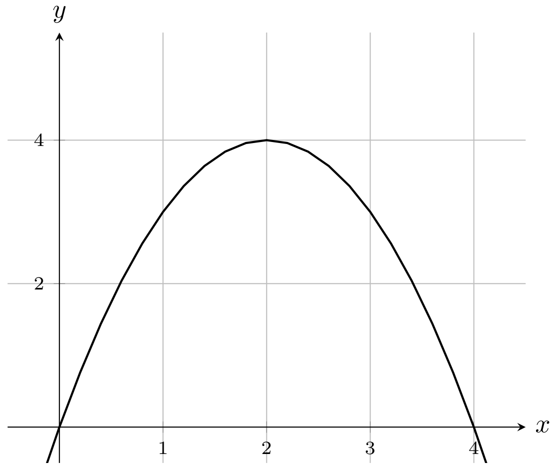
26. With three rectangles, approximate the area under $f(x) = \sin(x)^2$ between $x = 0$ and $x = \frac{3\pi}{2}$ using right endpoints.

27. Consider the given function $f(x)$ and the rectangles approximating its area on $[-3, 3]$. Give an expression for the area of the rectangles in the figure below. Use $\sum$ notation to express it.



28. Consider the given function $f(x) = -(x-2)^2 + 4$ on $[1, 4]$. Find an estimate of the area under the graph of f on $[1, 4]$ by using four rectangles. Use the left endpoints as sample points.

   (a) Draw the rectangles.



   (b) Find the area of the rectangles.

29. The table below gives velocity of a particle at 4 time points. Using this data, approximate the distance travelled by the particle using the left endpoints.

time (s)	0	1	2	3
velocity (ft/s)	1	1	3	4

30. Consider $\int_1^3 \frac{2-x}{4} dx$

   (a) Express this integral as the limit of a sum.

   (b) Evaluate the integral using a geometric argument.

31. Write down the Riemann sum for the integral $\int_2^4 x^2 dx$ using a right-endpoint-rule with equally-spaced partitions Δx_n.

32. Consider the integral $\int_0^3 \frac{x-2}{3} dx$

   (a) Using the left endpoint rule and three equally spaced points, approximate the integral with a sum .

   (b) Compute the exact value of the integral using a geometric argument.

33. Using 4 points approximate the integral $\int_0^{2\pi} x|\sin(x)|dx$ with right endpoints.

34. **??** For each of the following Riemann Sums, identify the definite integral it converges to (but don't evaluate the integral). In addition, identify whether the left-endpoint, right-endpoint or midpoint Riemann Sum has been used.

   (i)

$$\lim_{n\to\infty} \frac{12}{n} \sum_{j=0}^{n-1} e^{2x_j} \left(x_j - x_j^2\right), \tag{5}$$

   where $x_j = -4 + \frac{12j}{n}$.

   (ii)

$$\lim_{n\to\infty} \frac{9}{n} \sum_{j=1}^{n} \left(3x_j - x_j^4\right) \cos(x_j), \tag{6}$$

where $x_j = -3 + \frac{9j}{n}$.

(iii)

$$\lim_{n\to\infty} \frac{10}{n} \sum_{j=0}^{n-1} \left(3x_j - x_j^4\right) \log(x_j), \tag{7}$$

where $x_j = 3 + \frac{10j+5}{n}$.

35. Consider the function

$$f(x) = \begin{cases} 0 & x < 1 \\ 2 & 1 \leq x \leq 3 \\ 1/2 & 3 < x \leq 7 \\ 5 & 7 \leq x \end{cases}$$

Compute the following integrals.

(a) $\int_{-1}^{3} f(x)dx$

(b) $\int_{0}^{9} f(x)dx$

(c) $\int_{1/2}^{3/2} f(x)dx$

36. Compute the following definite integrals:

(a) $\int_2^4 (x^2 - x)\, dx$

(b) $\int_{-1}^1 (e^x - 3)\, dx$

(c) $\int_1^e \frac{1}{x}\, dx$

37. For each of the following Riemann Sums, identify the definite integral it converges to (but don't evaluate the integral). In addition, identify whether the left-endpoint, right-endpoint or midpoint Riemann Sum has been used.

(i)

$$\lim_{n\to\infty} \frac{12}{n} \sum_{j=0}^{n-1} e^{2x_j} \left(x_j - x_j^2\right), \tag{11}$$

where $x_j = -4 + \frac{12j}{n}$.

(ii)

$$\lim_{n\to\infty} \frac{9}{n} \sum_{j=1}^{n} \left(3x_j - x_j^4\right) \cos(x_j), \tag{12}$$

where $x_j = -3 + \frac{9j}{n}$.

(iii)

$$\lim_{n\to\infty} \frac{10}{n} \sum_{j=0}^{n-1} \left(3x_j - x_j^4\right) \log(x_j), \tag{13}$$

where $x_j = 3 + \frac{10j+5}{n}$.