MATH 1310-012, Fall 2017

Midterm #3

Instructions:

1. Check that you have all pages.
2. Write your name on the front page.
3. You have 50 minutes for this exam.
4. Write down all your work for full credit.
5. You are not allowed to use calculators or phones.

Name:_____

Question	Points	Score
Related Rates	20	
Shapes of Curves	20	
l'Hospitals rule	20	
Optimization	20	
Riemann sum	20	
Integrals	20	
Total:	120	

1. (20 points) **Related Rates** Suppose a sphere's radius is growing at a rate $2\,\frac{\text{cm}}{\text{min}}$. At what rate is the volume growing when the radius is 3 cm? Note: The volume inside a sphere is $V = \frac{4}{3}\pi r^3$.

2. (20 points) **Shapes of Curves** Consider the function $f(x) = 2x^4 - 8x^3 - 16x^2 + 12$.

   (a) Find the intervals, where f is increasing and the intervals where f is decreasing.

   (b) Find the local maxima and minima.

3. (20 points) **l'Hospitals rule** Identify the indeterminate form and evaluate the following limits using l'Hospitals rule.

   (a) $\lim_{x \to 1} \frac{e^{x^2-1} - 1}{x^2}$

   (b) $\lim_{x \to 0} \cos(x)^{\frac{1}{x}}$

4. (20 points) **Optimization** Suppose a soup manufacturer want's to stand out from their competition by packaging their new line of organic soups in rectangular cartons with a base depth d that is $\frac{3}{2}$-times its base width w. The soup volume is fixed at 600 ml (1 ml $= 1\,\text{cm}^3$). The rectangular carton shape must be chosen with a given height h, width w, and depth d in cm. In order to minimize packaging costs, the manufacturer wants to find the rectangular shape that minimizes surface area. Your goal, find the dimensions (in cm) of the minimum-surface area carton. You may leave your calculations for w, h, and d as algebraic expressions.

   (a) Write down equation for the volume V?

   (b) Write down function for surface area S?

   (c) Find the optimal box dimensions. Leave your answer as an algebraic expression.

5. (20 points) **Riemann sum**

(a) What integral does the following Riemann sum converge to?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{10}{n} \left(e^{x_i} - x_i \cos(x_i)\right)$$

where $x_i = -2 + \frac{10i}{n}$. Is this a left-endpoint or right-endpoint rule?

(b) Write down the right-endpoint Riemann sum approximation to the following integral using n sample points. Express your answer in terms of the points $x_i = -2 + \frac{6i}{n}$.

$$\int_{-2}^{4} \left(x^3 + \sin(x)\right) \mathrm{d}x$$

6. (20 points) **Integrals** Evaluate the following integrals.

(a) $\int \left(\sin(x) - x^2 + 4e^x\right) \mathrm{d}x$

(b) $\int_1^3 \left(\frac{2}{x} - x\right) \mathrm{d}x$