MATH 1310 - 009 — Final Exam

Name: ________________________

Date: 12/15/2017

Instructor: Ethan Levien

No phones, calculators, or notes! Remember to show all your work.

1. (20 points) Suppose lighthouse is located on an island 3 km away from the nearest point P on a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when it is 1 km from P?

2. (20 points) Identify the indeterminate form and evaluate the following limits using l'Hospitals rule.

   (a) $\lim_{x \to 0} \frac{\sin(x)}{\ln(x+1)}$

   (b) $\lim_{x \to 0} \cos(x)^{\frac{1}{\sin(x)}}$

3. (20 points) Consider the function $f(x)$ that is defined on the positive real line $(0, \infty)$

$$f(x) = xe^{-3x}$$

(a) Find the interval(s) on the positive real line where $f(x)$ is increasing and decreasing.

(b) Find the x-value(s), if any, where $f(x)$ has zero slope.

4. (30 points) Compute the following derivatives

(a) $\frac{d}{dx}\int_{-100}^{x^2} \frac{1}{t} dt$

(b) $\frac{d}{dx} e^{x^2+\frac{1}{x^2}}$

(c) $\frac{d}{dx}\left(\frac{\sin(2x)}{2+\sqrt{x}}\right)$

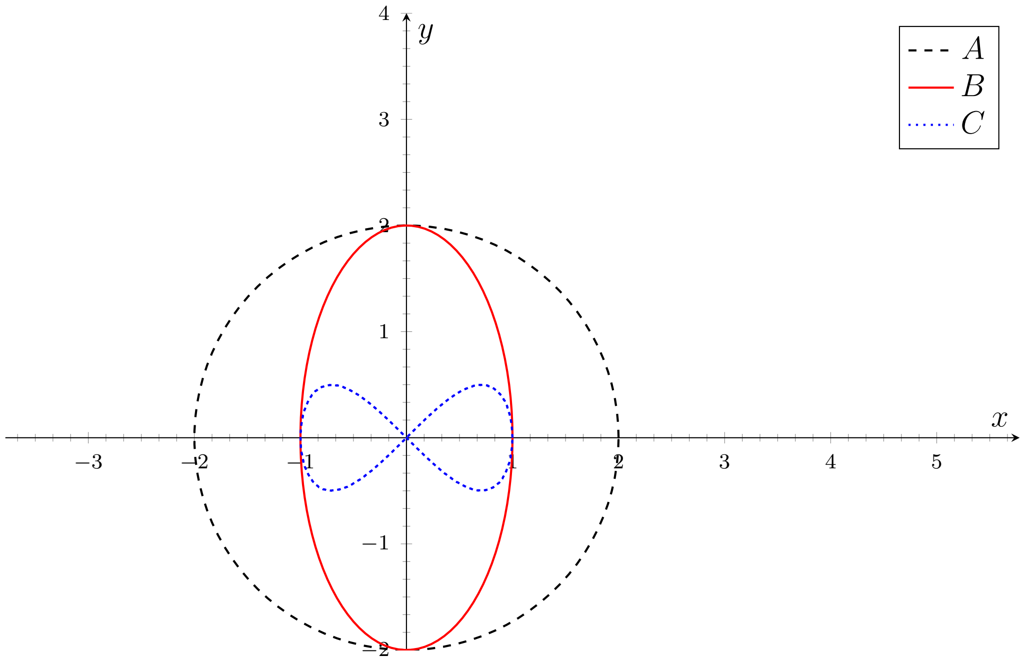
5. (20 points) Consider the curve defined by the equation $\sin(y)x = x^2$
   (a) Find an expression for $\frac{dy}{dx}$

   (b) Find the slope dy/dx of the tangent line to the curve at the point $(x, y) = (1, 0)$

6. (20 points) Consider the parametric equations

    1. $x = 2\cos(t), \quad y = 2\sin(t)$
    2. $x = \cos(t), \quad y = 2\sin(t)$
    3. $x = \cos(2t), \quad y = \frac{1}{2}\sin(4t)$

    where $0 \leq t \leq 2\pi$. Match each of them with the corresponding curve in the figure below. Explain your choice.



7. (30 points) Determine whether the following improper integrals converge or diverge and compute the value if they converge

   (a) $\displaystyle\int_0^\infty xe^{-10x^2}dx$

   (b) $\displaystyle\int_1^\infty \frac{\ln(x)}{x}dx$

8. (30 points) Compute the following definite and indefinite integrals

   (a) $\displaystyle\int 5\tan(10x^2)x\,dx.$

   (b) $\displaystyle\int \cos(x)x\,dx.$

   (c) $\displaystyle\int_0^1 \frac{10x}{(x^2+4)^2}dx.$

9. Find the global minimum point r^* of the function

$$L(r) = \sqrt{r^2 + \frac{16}{r^2}}, \quad for \;\; r > 0.$$

That is, $L(r^*) \leq L(r)$ for all $r > 0$.

10. (20 points) Compute the derivative of

$$f(x) = \frac{4x^2 - 100x}{x}$$

using the **limit-based definition**.

11. (20 points) Determine if the derivative of

$$f(x) = \begin{cases} 3x, & x < 3 \\ x^2, & x \geq 3 \end{cases}$$

exists at $x = 3$.

12. (20 points) Suppose the function $f(t)$ represents the velocity (meters/second) of an object for t in the range $[0, \sqrt{\pi/2}]$ seconds.

$$f(t) = t\cos(t^2)$$

(a) Write down the correct expression for the distance traveled starting from time $t = 0$ to ending time $t = \sqrt{\pi/2}$.

(b) Compute the exact value of the expression from (a).

13. (20 points) Find x and y such that $x + y = 6$ and xy^2 a maximized

Problem	Points	Score
1	20	
2	20	
3	30	
4	30	
5	20	
6	20	
7	20	
8	20	
9	30	
10	30	
11	20	
12	20	
13	20	
	Total:	