Final Exam

Math 1310-009 - Engineering Calculus I

December 16, 2015

8:00AM-10:00AM

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it will be considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

By signing below, you are acknowledging that you have read and agree to the above paragraph,

Name:______________________________

Signature:______________________________

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Question	Points	Score
1	15	
2	30	
3	15	
4	12	
5	20	
6	21	
7	15	
8	20	
9	15	
10	12	
11	10	
12	15	
13	10	
Total:	210	

Note: There are 210 total possible points to be earned, but the exam will be graded out of 200.

Advice: If a problem is giving you trouble, skip it and return later! Spending too much time on a problem will provide you too little time to do the whole exam. If you are stuck on a particular detail, write down how you would proceed through the problem. Good luck!

$\boxed{15}$ 1. (a) Using the definition of the definite integral of f from a to b, express the following limit of a Riemann sum as a definite integral on the given interval.

$$\lim_{n\to\infty}\sum_{i=1}^{n}[x_i^* - 1]\Delta x, \qquad [a,b]=[0,3]$$

(b) Using the Evaluation Theorem, find the net area under the curve given by $y = x - 1$ between $x = 0$ and $x = 3$.

(c) Check your answer in part (b) by sketching a graph of the curve given by $y = x - 1$ and using geometry to find the net area between $x = 0$ and $x = 3$.

$\boxed{30}$ 2. Evaluate the following integrals:

(a) $\int \frac{2x^2-5x+2}{x^3+x}dx$

(b) $\int \cos x \sin^4 x \, dx$

(c) $\int x^2 \ln x \, dx$

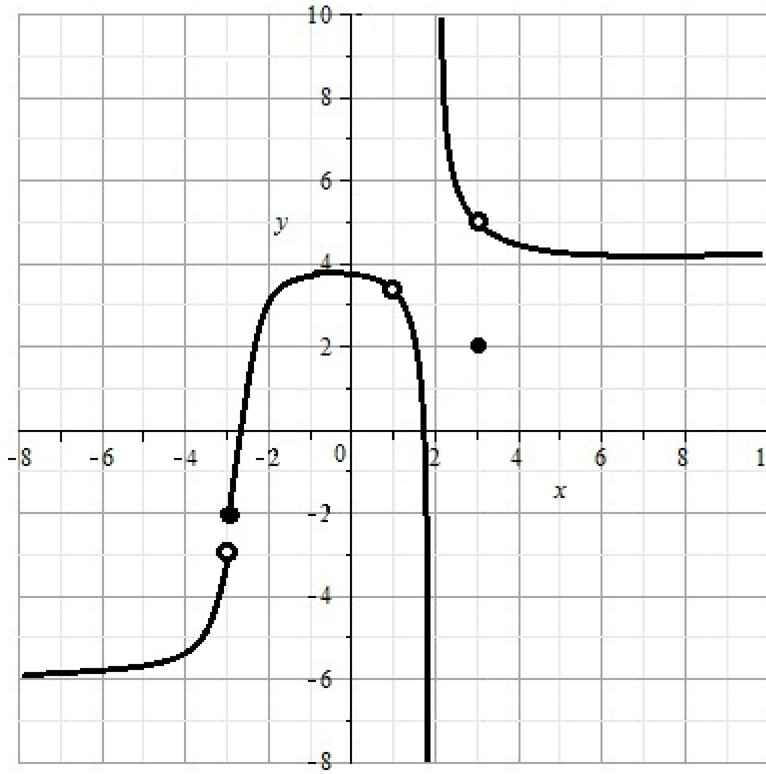
$\boxed{15}$ 3. Suppose an object moves along a line so that its velocity function during the time period $0 \leq t \leq \sqrt{\pi}$ is given by

$$v(t) = t\sin(t^2)$$

(a) Write down an expression to find the displacement of the object from time $t = 0$ to time $t = \sqrt{\pi}$.

(b) Evaluate the expression from part (a).

12 4. Consider the sketch of the graph of the function $f(x)$ given below and answer the following question:



Name the type of discontinuity that exists at each of the following values of x, and identify the condition(s) for continuity that fail(s) to be met:

- $x = 3$

- $x = 2$

- $x = -3$

20 5. Evaluate the limit, if it exists. In your answer, be sure to make a distinction between limits that are infinite and limits that do not exist.

(a) $\lim_{x\to 3} \frac{2(x-3)}{x^2-9}$

(b) $\lim_{x\to -\infty} \frac{3x^2+1}{2x+5}$

(c) $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

(d) $\lim_{x\to \infty} x^{1/x}$

$\boxed{21}$ 6. Compute the derivatives of the following functions. You do not need to simplify your answer.

(a) $f(x) = x^3 e^{4x}$

(b) $h(x) = \frac{1+\sin x}{x+\cos x}$

(c) $g(x) = \int_x^1 \cos\sqrt{t}\, dt$

$\boxed{15}$ 7. Use implicit differentiation to find an equation of the tangent line to the curve given by

$$x^2 + y^3 - 2y = 3.$$

at the point (2, 1).

20 8. Consider the function $f(x)$ given below whose domain is $(0, \infty)$.

$$f(x) = \ln(x) - \frac{x^2}{8}$$

(a) Find the x-value(s), if any, where f has a horizontal tangent line.

(b) Find the interval(s) where f is increasing or decreasing on its domain.

(c) Find the interval(s) of concavity and point(s) of inflection, if any, for f on its domain.

(d) Use the information from parts (a)-(c) to sketch a graph of $f(x)$. Be sure to represent intervals on which f is increasing or decreasing, concave up or down, and identify, if any, point(s) of inflection or critical number(s) of f.

Note: The two roots (or zeros) of the function $f(x) = \ln(x) - \frac{x^2}{8}$ are $x \approx 1.2$ and $x \approx 2.9$

15 9. Suppose a paper cup has the shape of a cone with height 16 cm and radius 8 cm (at the top). If water is being poured into the cup at a rate of 2 cm^3/s, how fast is the water level rising when the water is 4 cm deep? Be sure to include units in your answer.

Recall: The volume of a cone is given by: $V = \frac{1}{3}\pi r^2 h$.

$\boxed{12}$ 10. The displacement (in meters) of a particle moving in a straight line is given by the equation

$$s(t) = t^2 - 8t + 18$$

where t is measured in seconds.

(a) Find the average velocity over the time interval [3, 4]. Be sure to include units in your answer.

(b) Find the instantaneous velocity at $t = 4$ seconds. Be sure to include units in your answer.

(c) Find the value of c guaranteed to exist by the Mean Value Theorem such that the instantaneous velocity at c equals the average velocity over the time interval [3, 4].

$\boxed{10}$ 11. Use the definition of the derivative to compute $f'(t)$ given:

$$f(t) = t^2 - 8t + 18$$

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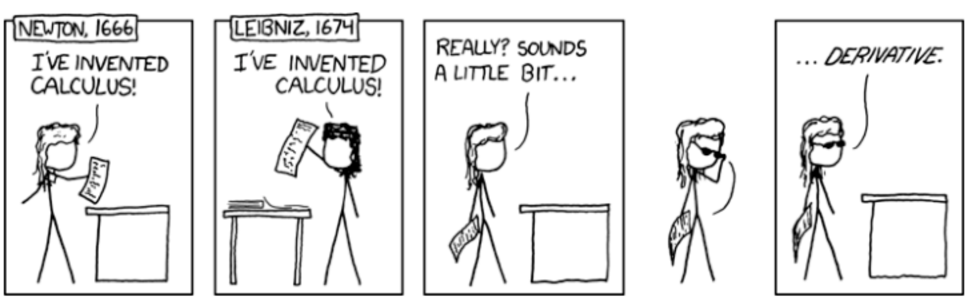

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15 12. Suppose a cylindrical can is designed to have a volume of $20\pi \text{cm}^3$. Heavier gauge (more expensive) metal, costing $\$10/\text{cm}^2$ is used to construct the top and bottom of the can while thinner (cheaper) metal, costing $\$8/\text{cm}^2$ is used for the body, or side, of the can. Find the dimensions r, h that minimize the cost of the can. Be sure to justify your answer as to why these dimensions give the cheapest can and include units in your answer.

Recall: The volume and surface area of a cylinder, respectively, are given by:

$$V = \pi r^2 h \qquad\qquad A = 2\pi r^2 + 2\pi r h$$

Note: You will have to modify one of these equations to account for the difference in the cost of the material used to construct the top and bottom as compared to the side of the cylindrical can in order to meet your objective. That is, you need to find the cost equation (in dollars),

$$C = \text{total cost of bottom and top} + \text{total cost of side}$$

$\boxed{10}$ 13. Determine whether the following improper integrals converge or diverge:

(a) $\int_1^\infty \frac{1}{x} dx$

(b) $\int_2^3 \frac{1}{\sqrt{3-x}} dx$