Math 1310-4, Fall 2017

FINAL EXAM-ALTERNATE

To receive partial credit you must justify your work (intermediate steps). Write clearly and highlight your final answer. If you run out of room, use the back of the page clearly indicating it.

No textbook, student-made formula sheet, calculator, computer, cell phone or mobile device are allowed.

By signing below you acknowledged having read the above instructions and that you will abide by the University Honor Code.

NAME: ______________________________

Unid: ______________________________

SIGNATURE: ______________________________

THINK WELL

Marks Breakdown

Question	Score
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
Total	

Some Trigonometric Identities

Derivative identities

$$\begin{aligned}
\frac{d}{dx}\arctan(x) &= \frac{1}{1+x^2} \\
\frac{d}{dx}\arccos(x) &= \frac{-1}{\sqrt{1-x^2}} \\
\frac{d}{dx}\arcsin(x) &= \frac{1}{\sqrt{1-x^2}} \\
\frac{d}{dx}\cos(x) &= -\sin(x) \\
\frac{d}{dx}\sin(x) &= \cos(x) \\
\frac{d}{dx}\sec(x) &= \sec(x)\tan(x) \\
\frac{d}{dx}\tan(x) &= \sec^2(x).
\end{aligned}$$

Some integral identities

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C. \int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}(x) + C.$$

Pythagorean Identities

$$\begin{aligned}
\cos^2(x) + \sin^2(x) &= 1 \\
\tan^2(x) + 1 &= \sec^2(x).
\end{aligned}$$

Double Angle Identities:

$$\begin{aligned}
\sin\big(A+B\big) &= \sin(A)\cos(B)+\cos(A)\sin(B) \\
\sin\big(A-B\big) &= \sin(A)\cos(B)-\cos(A)\sin(B) \\
\cos\big(A+B\big) &= \cos(A)\cos(B)-\sin(A)\sin(B) \\
\cos\big(A-B\big) &= \cos(A)\cos(B)+\sin(A)\sin(B) \\
\tan\big(A+B\big) &= \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)} \\
\sin(2A) &= 2\sin(A)\cos(A) \\
\cos(2A) &= \cos^2(A)-\sin^2(A).
\end{aligned}$$

The Evaluation of Trig Functions at Certain Angles.

Trig. Function	Angle in Radians	Value
cos	0	1
cos	$\frac{\pi}{6}$	$\sqrt{3}/2$
cos	$\frac{\pi}{4}$	$1/\sqrt{2}$
cos	$\frac{\pi}{3}$	$1/2$
cos	$\frac{\pi}{2}$	0
sin	0	0
sin	$\frac{\pi}{6}$	$1/2$
sin	$\frac{\pi}{4}$	$1/\sqrt{2}$
sin	$\frac{\pi}{3}$	$\sqrt{3}/2$
sin	$\frac{\pi}{2}$	1
tan	0	0
tan	$\frac{\pi}{6}$	$1/\sqrt{3}$
tan	$\frac{\pi}{4}$	1
tan	$\frac{\pi}{3}$	$\sqrt{3}$
tan	$\frac{\pi}{2}$	∞

First / second mid-term questions:

1. Consider the function $f(x) = 5\ln(1+\sqrt{3x-2}) = y$
   (a) Determine the domain $D =?$ and range $R =?$ of $f(x)$.
   (b) Find the inverse function $f^{-1}(y) = x$.

2. Evaluate the following limits. Use limit laws, and identify the indeterminate form and use L'Hospital's rule if necessary.
   (a) * $\lim_{x\to 2} \frac{x^2-x-2}{3x-6}$
   (b) $\lim_{t\to\infty} te^{-4t}$
   (c) $\lim_{x\to 0} \big(1+\sin(2x)\big)^{\frac{2}{x}}$

3. Compute the following derivatives using the appropriate derivative rules (do not use limit-based differentiation procedure).
   (a) $\frac{d^2}{dx^2}[x^2 + \frac{1}{x} + 5\sqrt{x}]$.
   (b) $\frac{d}{dx}[e^{x^2}]$.
   (c) $\frac{d}{dx}[x\sin(x)]$.
   (d) $\frac{d}{dx}\left(\frac{\sin(2x)}{2+\sqrt{x}}\right)$.

4. Consider the equation defining a curve in the x-y-plane

$$\frac{x^2}{4} + xy^2 = 3.$$

Find the slope of the tangent line of the curve at the point $(x, y) = (2, 1)$.

5. * Consider the function $f(x)$ that is defined on the positive real line $(0, \infty)$

$$f(x) = xe^{-5x}$$

(a) Find the interval(s) on the positive real line where $f(x)$ is increasing and decreasing.

(b) Find the x-value(s), if any, where $f(x)$ has zero slope.

Third mid-term questions:

6. * Find the global minimum point r^* of the function

$$L(r) = \sqrt{r^2 + \frac{4}{r^2}}, \quad for \ \ r > 0.$$

That is, $L(r^*) \leq L(r)$ for all $r > 0$. Briefly justify why your chosen point is the global minimum.

7. Two toy remote-controlled cars start from the same position at time $t = 0$. One travels north at 4 m/s, while the other travels west at 3 m/s. After two seconds, compute the rate of change of the linear distance between the two cars. Draw a diagram to help you answer the question and clearly state the units. Make sure that you simplify your answer as much as possible.

8. (a) Let A be the point with Cartesian coordinates $(1, 4)$ Find an expression $S(y)$ for the distance between A and a point (x, y) on the parametric curve $y^2 - 2x = 0$. Your expression must be wholly in terms of y, not x.

   (b) Find the point on the curve that is closest to A. Briefly justify your answer.

9. (a) Consider the Riemann sum

$$\lim_{N\to\infty} \frac{13}{N} \sum_{j=1}^{N} x_j^3 \cos(2x_j),$$

where $x_j = -4 + \frac{13j}{N}$. Write down the correct integral with correct limits of integration $\int_?^? ?dx$ that equals the above Riemann sum

(b) Write down the N-point Riemann sum of the integral $\int_0^{10} xe^{x^2}\,dx$, that uses the right-endpoint rule. Express your summation in terms of the points $\{y_j\}$, where $y_j = \frac{j}{N}$.

Integration / Fundamental Theorem of Calculus questions:

10. (a) Compute the following integral
$$\int_0^1 \frac{10x}{(x^2+4)^2}\,dx.$$

(b) Suppose the function $f(t)$ represents the velocity (meters/second) of an object for t in the range $[0, 2]$ seconds.
$$f(t) = t\cos(t^2)$$

(a) Write down the correct expression for the distance traveled starting from time $t = 0$ to ending time $t = 2$.

(b) Compute the exact value of the expression from (a).

11. Compute the derivative with respect to x of

(a)
$$F(x) = \int_{-200}^{x} \cos(t)dt.$$

(b)
$$F(x) = \int_{x}^{x^2} \sin(t)dt$$

12. Compute the following integrals
    (a) $\int_1^4 3x^2 + \frac{1}{x} + \sqrt{x}dx$
    (b) $\int xe^{2x}dx$
    (c) $\int \cos(x)e^{2x}dx$. (Hint: Use integration by parts twice)

13. Compute the following integrals
    (a) $\int \frac{x+2}{x^2-5x+6} dx$
    (b) $\int \frac{2-3x}{x^3+4x}$

14. Compute the following integrals
    (a) $\int_0^{\frac{\pi}{6}} \sin^3(2x)dx$.
    (b) $\int \frac{dx}{x^2\sqrt{1-4x^2}}$

15. Do the following improper integrals converge or diverge? Briefly justify your answer. If the integral converges, then compute what it converges to.

(a)

$$\int_0^\infty xe^{-4x}dx$$

Hint: You might want to use your answer in question (2b).

(b)

$$\int_{\frac{1}{2}}^5 \frac{dx}{\sqrt{2x-1}}.$$

16. Does the following integral converge or diverge?

$$\int_0^{\pi} \frac{100\sin^2 x}{\sqrt{x}} dx$$

Hint: Use the Comparison Theorem, and properties of the integral $\int_0^{\pi} \frac{1}{\sqrt{x}} dx$.