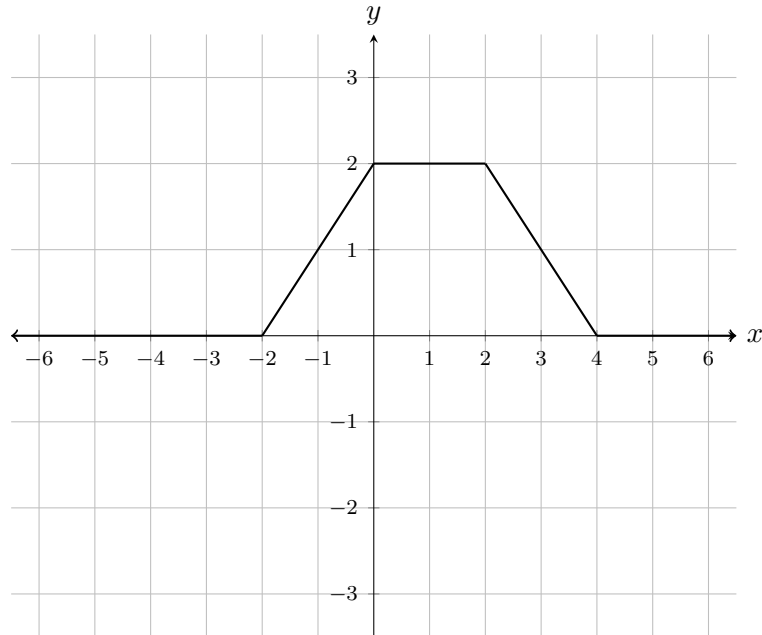


Midterm exam 1

1. Function transformation. Consider the function $f(x)$ depicted in the graph. Draw a graph of the transformed function

$$-af(x+b)$$

on the same axes.



2. Specify the domain and range of $f(x) = \ln(100 - x^2)$.
3. Find the equation $y = mx + b$ for the secant line of the function $f(x) = x^2 + 3x - 1$ that passes between points $x = 0$ and $x = 1$.
4. Find the inverse function $f^{-1}(y) = x$ of $f(x) = y = \ln\left(\frac{x}{1-x}\right)$. Determine the domain and range of f and f^{-1} .

Solution:

$$e^y = \frac{x}{1-x}$$

so

$$(1-x)e^y = x$$

so

$$e^y = x + xe^y$$

so

$$x = \frac{e^y}{1+e^y}$$

from which we deduce

$$f^{-1}(y) = \frac{e^y}{1+e^y}.$$

5. Limits: Compute the following limits, or explain why the limit does not exist.

(a) $\lim_{x \rightarrow 5} \frac{|x-5|}{x-5}$.

(b) $\lim_{x \rightarrow -2} \frac{x^2+3x+2}{x^2-4}$.

(c) $\lim_{x \rightarrow \infty} \frac{2x^2+1}{x^2-4}$.

6. Consider the parametric equations

(a) $x = \cos(t), \quad y = \sin(t) + 1$

(b) $x = 2 \cos(t) + 1, \quad y = 2 \sin(t)$

(c) $x = \cos(2t), \quad y = \frac{1}{2} \sin(4t)$

Match each of them with the corresponding curve in the figure below. Explain your choice.

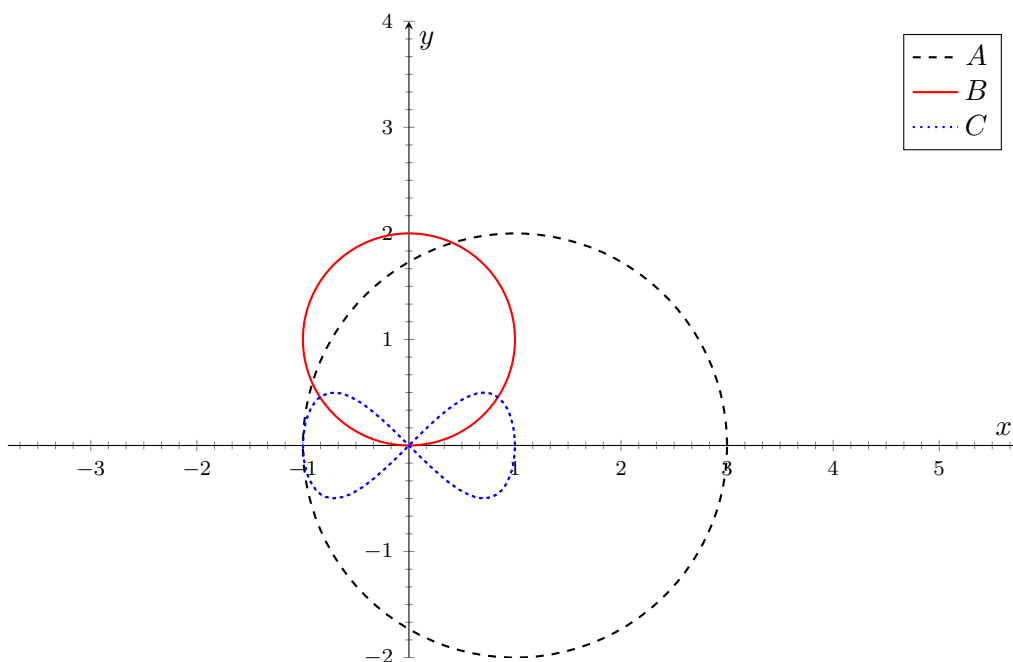


Figure 1: