MATH 1310-4 — Midterm 1 Fall 2017

Name and Unid: ______________________________

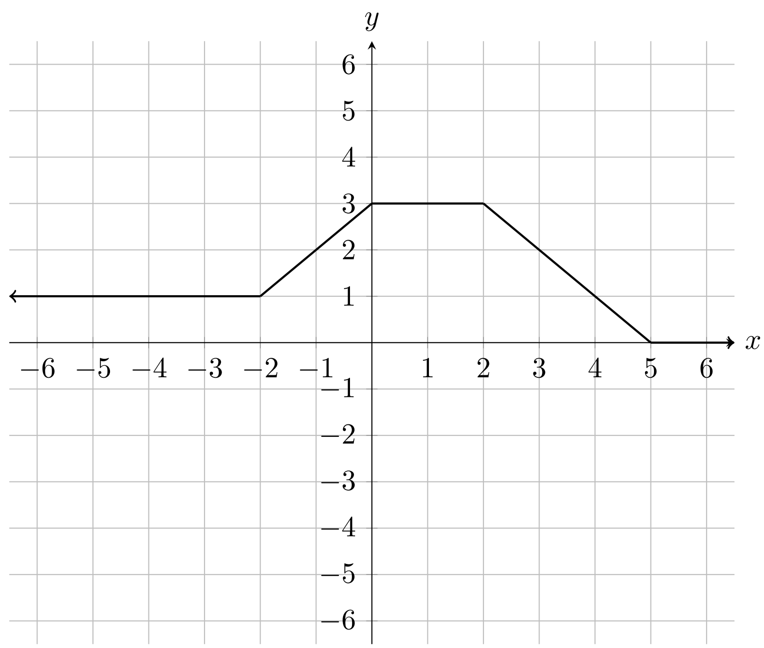
Date: 09/15/2017

Instructor: William Nesse
No phones, calculators, or notes. Show all your work for full credit.

1. (20 points) Function transformation. Consider the function $f(x)$ depicted in the graph. Draw a graph of the transformed function

$$-2f(x+3)$$

on the same axes.



2. (30 points) Compute the following limits, or explain why the limit does not exist.

   (a) $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$.

   Solution:
   $$\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4} = \lim_{x \to 2} \frac{(x-2)(x+3)}{(x-2)(x+2)} = \lim_{x \to 2} \frac{x+3}{x+2} = \frac{5}{4}.$$

   (b) $\lim_{x \to -2} \dfrac{x + 2}{|x + 2|}$.

   Solution:
   $$\lim_{x \to -2^+} \frac{x+2}{x+2} = 1$$
   $$\lim_{x \to -2^-} \frac{x+2}{-x-2} = -1$$
   the one-sided limits do not agree so the limit does not exist.

   (c) $\lim_{x \to \infty} \dfrac{x^3 + \frac{2}{x^2}}{2x^3 + x}$

   Solution: One must transform the expression by multiplying top and bottom by one over the leading power:
   $$\lim_{x \to \infty} \left(\frac{x^3 + \frac{2}{x^2}}{2x^3 + x} \right) \left(\frac{\frac{1}{x^3}}{\frac{1}{x^3}} \right)$$
   $$= \lim_{x \to \infty} \frac{1 + \frac{2}{x^5}}{2 + \frac{1}{x^2}} = \frac{1+0}{2+0} = \frac{1}{2}$$

3. (20 points) Find the equation of the secant line $y = mx + b$ of the function $f(x) = x + \frac{1}{x}$ that passes between points $x = \frac{1}{2}$ and $x = 1$.

Solution: The secant line goes through the points $(1/2, f(1/2)) = (1/2, 3/2)$ and $(1, f(1)) = (1, 2)$. Then the slope is

$$m = \frac{2 - (3/2)}{1/2} = 1$$

Using the point-slope formula

$$1 \cdot 1 + b = 2 \implies y = x + 1$$

4. (20 points) Find the inverse function $f^{-1}(y) = x$ of

$$f(x) = y = \ln\left(\frac{2x}{x-1}\right).$$

Solution:

$$e^y = \frac{2x}{x-1}, \qquad (x-1)e^y = 2x, \qquad xe^y - e^y = 2x, \qquad x(e^y - 2) = e^y$$

$$x = \frac{e^y}{e^y - 2}$$

so,

$$f^{-1}(y) = \frac{e^y}{e^y - 2}.$$

5. (20 points) Specify the domain and range of $f(x) = 5\ln(9 - x^2)$.

Solution: $\ln(x)$ is defined for positive numbers, so f is defined for

$$0 < 9 - x^2, \qquad x^2 < 9, \qquad -3 < x < 3.$$

So the domain is $(-3, 3)$. For the range: $9 - x^2$ inputs $(0, 9)$ into log. So the range of f is $(-\infty, 5\ln(9)) = (-\infty, 10\ln(3))$.

6. (20 points) Consider the parametric equations defined for the time interval $0 \le t \le 2\pi$.

   1. $x(t) = 2\cos(t), \quad y(t) = 2\sin(t)$
   2. $x(t) = 2\cos(\frac{t}{2}), \quad y(t) = 2\sin(t)$
   3. $x(t) = \cos(t) + 2, \quad y(t) = \sin(t) - 1$

   Match each of them with the corresponding curve in the figure below. Explain your choice.

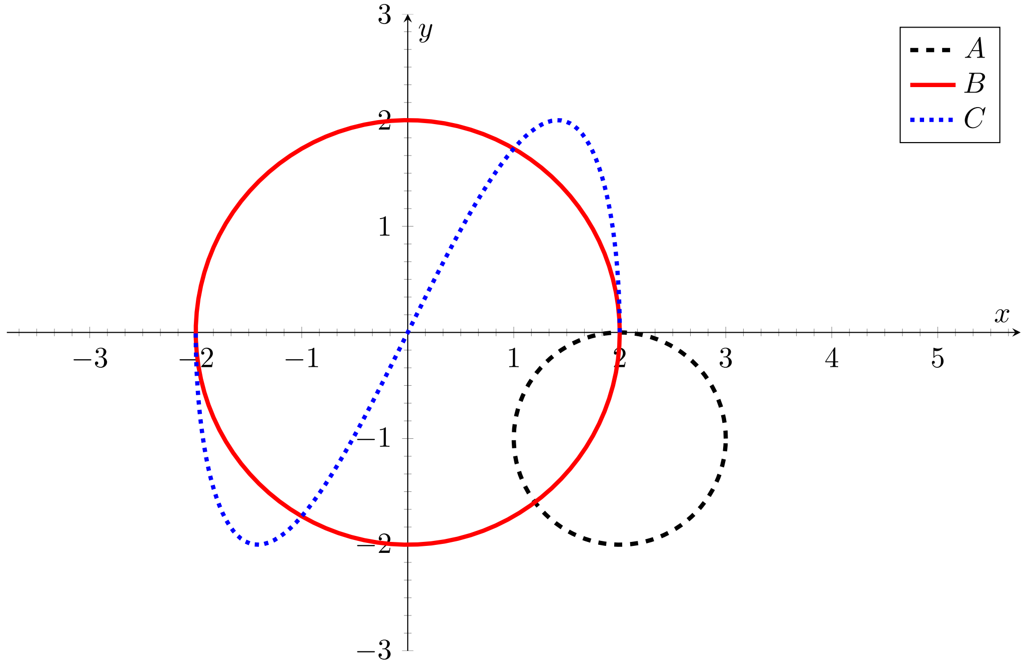


Figure 1

Solution: A and B are clearly circles.

1= B,

2 = C,

3= A.