Math 1310-1, Fall 2019

FINAL EXAM

To receive partial credit you must justify your work (intermediate steps). Write clearly and highlight your final answer. If you run out of room use another page and clearly indicate where.

No textbook, student-made formula sheet, calculator, computer, cell phone or mobile device are allowed.

By signing below you acknowledged having read the above instructions and that you will abide by the University Honor Code.

NAME: ________________________________

Unid: ________________________________

SIGNATURE: ________________________________

THINK WELL

Trigonometric Identities

Derivative identities

$$\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$$
$$\frac{d}{dx}\cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$

Some integral identities

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$
$$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}(x) + C$$

Pythagorean Identities

$$\cos^2(x) + \sin^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$

Double Angle Identities:

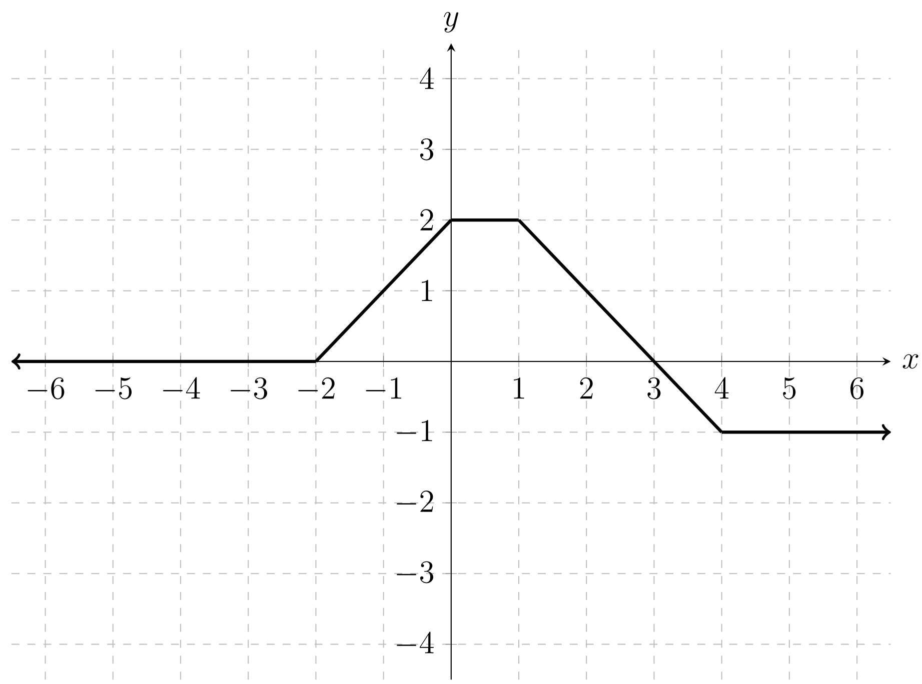
$$\sin\big(A+B\big) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\sin\big(A-B\big) = \sin(A)\cos(B) - \cos(A)\sin(B)$$
$$\cos\big(A+B\big) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\cos\big(A-B\big) = \cos(A)\cos(B) + \sin(A)\sin(B)$$
$$\tan\big(A+B\big) = \frac{\tan(A)+\tan(B)}{1-\tan(A)\tan(B)}$$
$$\sin(2A) = 2\sin(A)\cos(A)$$
$$\cos(2A) = \cos^2(A) - \sin^2(A)$$

Trig Functions at Certain Angles.

Trig. Function	Angle in Radians	Value
cos	0	1
cos	$\frac{\pi}{6}$	$\sqrt{3}/2$
cos	$\frac{\pi}{4}$	$1/\sqrt{2}$
cos	$\frac{\pi}{3}$	$1/2$
cos	$\frac{\pi}{2}$	0
sin	0	0
sin	$\frac{\pi}{6}$	$1/2$
sin	$\frac{\pi}{4}$	$1/\sqrt{2}$
sin	$\frac{\pi}{3}$	$\sqrt{3}/2$
sin	$\frac{\pi}{2}$	1
tan	0	0
tan	$\frac{\pi}{6}$	$1/\sqrt{3}$
tan	$\frac{\pi}{4}$	1
tan	$\frac{\pi}{3}$	$\sqrt{3}$
tan	$\frac{\pi}{2}$	∞

(20 pts) 1. Consider the function $f(x)$ depicted in the graph. Draw a graph of the transformed function on the same axes.

$$-2f(x+3).$$



(20 pts) 2. Consider the function $f(x) = \frac{x+2}{x^2+1}$.

(a) Find the secant line $y = mx + b$ through the points $x = 0$ and $x = 2$.

(b) Use the limit-based-definition of the derivative to find the line $y = mx + b$ tangent to the curve at $x = 0$.

(40 pts) 3. Compute the following derivatives using the appropriate derivative rules (do not use the limit-based differentiation procedure). Clearly delineate your work for each subpart.

(a) $\frac{d}{dx}[e^{-x^2}]$.

(b) $\frac{d}{dx}[x^2\cos(x)]$.

(c) $\frac{d}{dx}[\ln(\tan^{-1}(x))]$.

(d) $\frac{d}{dx}\left(\frac{xe^x}{2+\sqrt{x}}\right)$.

(20 pts) 4. Consider the equation for an ellipse

$$3x^2 + \frac{y^2}{9} = 4.$$

Find the slope of the tangent line y' to the ellipse at the point $(x, y) = (1, -3)$.

(20 pts) 5. Consider the function $f(x)$ defined on the positive real line $[0, \infty)$

$$f(x) = xe^{-2x}$$

(a) Find the interval(s) on the positive real line where $f(x)$ is increasing and decreasing.

(b) Find the x-value(s), if any, where $f(x)$ has zero slope.

(c) Find the point(s) of inflection, if any, for $f(x)$ and regions of the positive real line where the function is concave up and down.

(d) Based on the results from above, sketch a graph of $f(x)$, indicating the intervals and points in (a)-(c), correctly represent increasing and decreasing regions, and concavity.

(30 pts) 6. Evaluate the following limits. Use limit laws, and identify the indeterminate form in order to use L'Hospital's rule if necessary.

(a) $\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sin(x)}\right)$.

(b) $\lim_{x\to -1}\frac{\ln(x+2)}{x+1}$.

(c) $\lim_{x\to 0}[\cos(x)]^{\frac{1}{x}}$.

7. A store wants to decide at what price x to sell a popular holiday toy. Let the function $T(x)$ be the number toys sold at a given price. The store predicts that 2200 toys will sell if priced at $x = 8$ dollars, and only 2000 will sell when the price increases to $x = 10$ dollars. Assume the price-sales relationship is linear $T(x) = mx + b$, so that any one-dollar price increase results in 100 less toys sold.

   Let the revenue $R(x)$ be x times $T(x)$: $R(x) = xT(x)$.

   (a) Write down an expression for $T(x)$ and $R(x)$.

   (b) Find the price x that maximizes total revenue $R(x)$.

(20 pts) 8. Consider the Riemann sum

$$\lim_{N\to\infty} \sum_{n=1}^{N} (\tfrac{n}{N^2}) e^{-2\frac{n}{N}}$$

Write down an integral with correct limits of integration $\int_?^? ?dx$ that equals the above Riemann sum. Do so by identifying all of the following: Δx_n partition widths, the representative partition x_n points, the function to be integrated $f(x)$, and the limits of integration.

(20 pts) 9. Compute the derivative $\frac{dF}{dx}$ given

$$F(x) = \int_2^x \frac{e^{-s}}{s^2 + \sin(s)} ds.$$

(20 pts) 10. Suppose the function

$$v(t) = te^{-2t},$$

represents the velocity (measured in meters/second) of an object at time $t \geq 0$. Compute the distance that the object travels starting at time $t = 0$ and ending at time $t = 10$.

(40 pts) 11. Compute the following four integrals (a-d)—two more are on the next page.

(a) $\int_1^3 x + \frac{1}{x} + \sqrt{x}\ dx$

(b) $\int_0^1 5x\sqrt{3x^2+2}\ dx$

(c) $\int x\cos(x)dx$

(d) $\int \frac{x}{x^2-4x+8}\ dx$

(20 pts) 12. Consider the solid of revolution constructed by revolving an area around the vertical y-axis. The area in the x-y-plane is defined as the area enclosed by $\sin(x) = y$ and the horizontal line $y = 0$. Set up the integral that computes the volume of the solid of revolution using the method of cylindrical shells, but do not compute the integral.