# MATH 1310 - 001 — Midterm 2

**Name**: _____ **Unid**: _____

**Date**: 11/5/2021

**Instructor**: William Nesse, PhD

Special accommodation exam for Troy Lefteris u1016648@utah.edu. Duration: 55 minutes.
To be taken Nov 22-24, 2021. No phones, calculators, or external notes. Show your work and reasoning
Answers can be left as algebraic expressions.

1. (20 points) Consider the curve defined by the equation

$$g(x, y) = xy - e^y = 0.$$

   (a) Find an expression for $\frac{dy}{dx}$ in terms of $x$ and $y$.

   (b) Demonstrate that the points $(x, y) = (e, 1)$ and $(x, y) = (-e^{-1}, -1)$ both lie on the curve defined by the equation $g(x, y) = 0$.

   (c) Determine the value the slope $\frac{dy}{dx}$ of the tangent line to the curve for both points in (b).

2. (20 points) Use the linear approximation of $f(x) = x - \ln(x) - 2$ starting with a guess of $x_0 = e$, to estimate the value of the root of the equation $f(x) = 0$. That is, do one iteration of Newton's method to find $x_1 =$?. Leave your answer as an algebraic expression involving sums and fractions. Show all work.

3. Suppose the volume $V(t)$ of a cube is growing at a rate 3 $\text{cm}^3$/min. Suppose at a certain time $t$, the area $a(t)$ of a single side of the cube is 4 $\text{cm}^2$. At what rate is $a(t)$ growing at that time $t$?
   (a) Find the functional relationship between $V(t)$ and $a(t)$.
   (b) Determine which of the values of $V(t)$, $a(t)$, $\frac{dV}{dt}$, $\frac{da}{dt}$ are given in the problem, and which ones must be solved for.
   (c) Solve for the remaining values you found in (b) using algebra and the techniques related rates.

4. Consider the function $f(x) = \frac{1}{(2-4x)^2+1}$. Find the intervals where $f$ is increasing and the intervals where $f$ is decreasing, and find all local extrema.

5. (20 points) Identify the indeterminate forms and evaluate the following limits using l'Hospitals rule (LHR). Be sure to verify the hypotheses permitting LHR before it is used, and be sure to write all "lim" symbols at each juncture.

   (a) $\lim_{x \to 1} \dfrac{\ln(x)}{x-1}$

   (b) $\lim_{x \to \infty} \left(1 + \dfrac{1}{x^2}\right)^x$.