MATH 1310 - 001 — Midterm 2

Name: ______________________ **Unid**: ______________________

Date: 11/5/2021

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No phones, calculators, or notes. Show your work and reasoning. Answers can be left as algebraic expressions.

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1. (20 points) Consider the curve defined by the equation

$$g(x, y) = xy - \ln(y) = 0.$$

   (a) Find an expression for $\frac{dy}{dx} = (??)$ in terms of x and y.

   (b) Verify that the points $(x, y) = (e^{-1}, e)$ and $(x, y) = (0, 1)$ both lie on the curve defined by the equation $g(x, y) = 0$.

   (c) Determine the value of $\frac{dy}{dx}$, the slope of the tangent line to the curve for both points in (b).

2. (20 points) Use the linear approximation of $f(x) = \sqrt[4]{x}$ to estimate the value of $f(18) = \sqrt[4]{18}$. Leave your answer as an expression involving sums and fractions of whole numbers. Hint: an easy-to-compute nearby point is $\sqrt[4]{16}$. Show your work.

3. Suppose the surface area $A(t)$ of a cube is growing at a rate 3 cm^2/min. Suppose at a certain time t, the surface area is 24 cm^2. At what rate is the length of the cube's sides $\ell(t)$ growing at that time t?
   (a) Find the functional relationship between $A(t)$ and $\ell(t)$.
   (b) Determine which of the values of $A(t)$, $\ell(t)$, $\frac{dA}{dt}$, $\frac{d\ell}{dt}$ are given in the problem, and which ones must be solved for.
   (c) Solve for the remaining values you found in (b) using algebra and the techniques related rates.

4. Consider the function $f(x) = e^{-(3x+2)^2}$. Find the intervals where f is increasing and the intervals where f is decreasing, and find all extrema (min/max points).

5. (20 points) Identify the indeterminate forms and evaluate the following limits using l'Hospitals rule (LHR). Be sure to verify the hypotheses permitting LHR before it is used, and be sure to write all "lim" symbols at each juncture.

   (a) $\lim_{x \to 1} \frac{\ln(x)}{x-1}$

   (b) $\lim_{x \to \infty} \left(\frac{1}{1+x^2} \right)^x$