

uid number: _____

Instructor: Kelly MacArthur

Instructions:

- Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown.
- All answers should be completely simplified, unless otherwise stated.
- There are no calculators or any sort of electronics allowed on this exam. Make sure all cell phones are put away and out of sight. If you have a cell phone out at any point, for any reason, you will receive a zero on this exam.
- You will be given an opportunity to ask clarifying questions about the instructions at exactly 8:35 a.m. (for a couple minutes). The questions will be answered for the entire class. After that, no further questions will be allowed, for any reason.
- You must show us your U of U student ID card when finished with the exam.
- The exam key will be posted on Canvas by noon.
- You are allowed to use one 4x6 inch note card for your reference during the exam.
- Important hint that you might need somewhere on the test:
$$x^3 + 3x^2 - 4 = (x + 2)^2(x - 1)$$

(This exam totals 105 points, with no specified extra credit problem. Your score will be considered as out of 100, which means the extra credit points are built into the test scores.)

1. Find the indicated derivative of the given functions.

(a) (10 points) $D_x \left(\frac{\tan(x^2 - \sqrt{x})}{\sin^3(2x)} \right)$ (Do **not** simplify!)

Answer 1(a): _____

(b) (15 points) Find $\frac{dy}{dx}$ given $x^2 y^2 - \cos(2xy) - \sin x - 4x^3 = 7$

(Solve for $\frac{dy}{dx}$, i.e. get it by itself, but don't simplify any further.)

Answer 1(b): _____

(c) (10 points) $f'''(x)$ for $f(x) = (8x+5)^{\frac{3}{4}}$

Answer 1(c): _____

2. (15 points) Do ONE of these two problems, A or B. If you attempt both, you MUST specify which one will get graded. We will not grade both of them and if you make us choose which to grade, we will not choose in your favor (we'll do whatever's easiest to grade).

A. As sand leaks out of a hole in a container, it forms a conical pile whose altitude (or height) is always the same as its radius. If the height of the pile is increasing at a rate of 6 inches per minute, find the rate at which the sand is leaking out when the altitude is 10 inches.

B. Suppose a spherical snowball is melting and the radius is decreasing at a constant rate, changing from 12 inches to 8 inches in 45 minutes. How fast is the volume changing when the radius is 10 inches?

Answer 2A: _____

Answer 2B: _____

3. For $f(x) = \frac{1}{4}x^4 + x^3 - 4x$, answer the following questions.

(a) (6 points) Fill in the sign line for $f'(x)$.

$f'(x)$ <----->

(b) (4 points) Find all local min and max **point(s)**, if they exist.

Max point(s): _____

Min point(s): _____

(c) (6 points) Fill in the sign line for $f''(x)$.

<-----> $f''(x)$

(d) (4 points) Find all inflection **point(s)**, if any exist.

inflection point(s): _____

(Note: This is #3 continued.)

For $f(x) = \frac{1}{4}x^4 + x^3 - 4x$, answer the following questions.

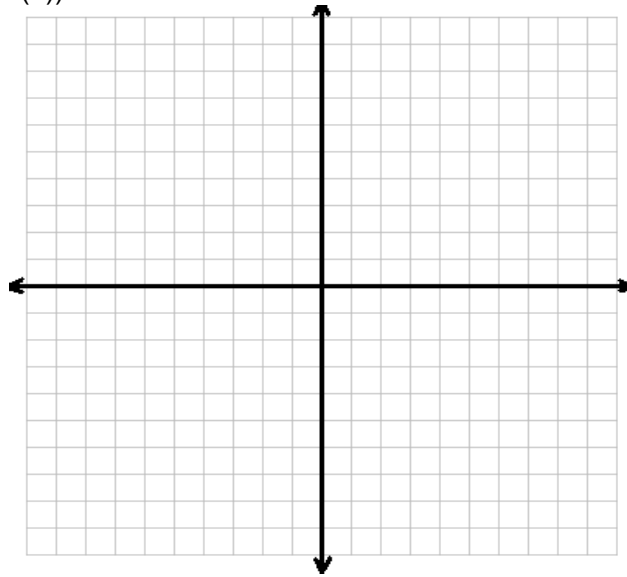
(e) (6 points) Identify the absolute (or global) max/min points on the closed interval $[0, 2]$ and explain how we know we're guaranteed absolute min/max points in this interval.

Absolute/global max point on $[0, 2]$: _____

Absolute/global min point on $[0, 2]$: _____

Why are we guaranteed global min/max points on this interval?

(f) (8 points) Sketch the **entire** graph of the function using **all** this information (from parts (a) through (e)).



4. (10 points) Let $y = 2(x-4)^3 - 3\sqrt[3]{x+3} + \sin\left(\frac{\pi x}{2}\right)$. If x changes from 5 to 4.999, approximately how much does y change?

Answer : _____

5. (11 points) Answer these short questions.

(a) Evaluate $\cos\left(\frac{7\pi}{3}\right)$. Answer: _____

(b) True or False (circle one)

The derivative of a product of two factors (where each factor is a function of x) is the product of the derivatives of each factor.

(c) True or False (circle one)

If $f'(c)$ exists, then $f(x)$ is continuous at $x=c$.

(d) Which of these statements are correct? (circle all that are correct)

(i) If $f'(a) = 0$, then there is a stationary point at $x = a$.

(ii) If there is a stationary point at $x = a$, then $f'(a) = 0$.

(iii) If $x = a$ makes the denominator zero in the original function, then there is a vertical asymptote at $x = a$.

(iv) If $x = a$ is a vertical asymptote for the graph of the function $y=f(x)$, then the function is undefined at $x = a$.

(e) Describe the three geometric possibilities that may occur in the graph of $y=f(x)$ at $x = a$, if we know that $f'(a)$ is undefined.

(i) _____

(ii) _____

(iii) _____