

uid number: _____

Seat #: _____

Instructor: **Kelly MacArthur**

Instructions:

- Please show all of your work as partial credit will be given where appropriate, **and** there may be no credit given for problems where there is no work shown.
- All answers should be completely simplified, unless otherwise stated.
- There are no calculators or any sort of electronics allowed on this exam. Make sure all cell phones are put away and out of sight. If you have a cell phone out at any point, for any reason, you will receive a zero on this exam.
- You will be given an opportunity to ask clarifying questions about the instructions at exactly 8:35 a.m. (for a couple minutes). The questions will be answered for the entire class. After that, no further questions will be allowed, for any reason.
- You must show us your U of U student ID card when finished with the exam.
- The exam key will be posted on Canvas by noon.
- There is a blank page attached to the back of this test. You may use this as scratch paper. You may use **NO** other scratch paper. Please transfer all finished work onto the proper page in the test for us to grade there. We will not grade the work on the scratch page.
- You are allowed to use one 4x6 inch note card for your reference during the exam.

(This exam totals 100 points, not including the extra credit problem.)

1. (5 points each) Find each limit, if it exists.

(a) $\lim_{x \rightarrow -1} (10x^5 - 3x^2 + 5x^{-1} - 7)$

Answer (a): _____

(b) $\lim_{x \rightarrow -3} \frac{2x^2 + x - 15}{4x + 12}$

Answer (b): _____

(c) $\lim_{x \rightarrow -\infty} \frac{4x^5 + x^3 - 7x + x^2}{9 + 11x + 9x^3 - 8x^5}$

Answer (c): _____

$$(d) \lim_{x \rightarrow -\infty} \frac{-5x^{\frac{8}{3}} + 6x - 27}{5x^2 + 6x}$$

Answer (d): _____

$$(e) \lim_{x \rightarrow 4} \frac{(x-3)(x+2)}{x^2-16}$$

Answer (e): _____

$$(f) \lim_{x \rightarrow 0} \frac{\sin(2x) \tan(x) - x \cos(4x)}{3x} \quad (\text{Hint: Separate into two fractions first.})$$

Answer (f): _____

2. (12 points) Find the equation of the tangent line to the curve $f(x) = 2x^4 - 5x^3 + 9x$ at $x = 1$.

tangent line: _____

3. (12 points) For $f(x) = \frac{-x^2(x+2)(x-5)}{6x(3x+2)(x-5)(x-2)}$,

(a) Find the x-values where $f(x)$ is discontinuous and categorize the type of discontinuity. (Make sure you show some work to support your claim.)

f(x) discontinuous at x = _____
 where It's a _____ hole jump vertical asymptote (circle one)

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f(x) discontinuous at x = _____
 where It's a _____ hole jump vertical asymptote (circle one)

(b) For what x-value(s) is this function "patchable?"

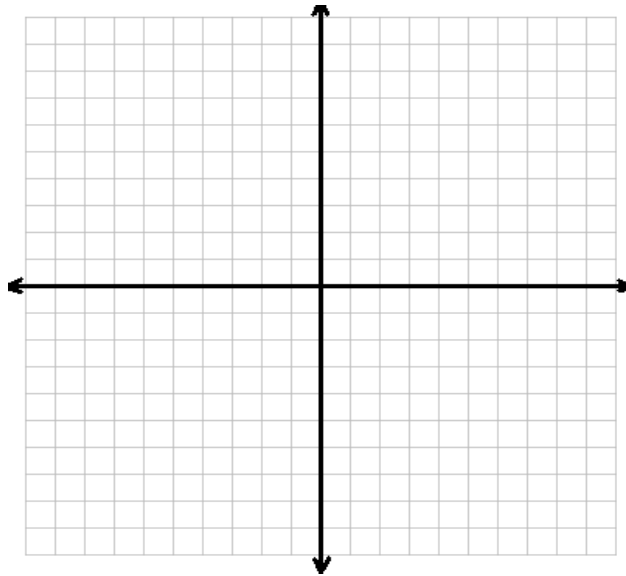
4. (15 points) Use the definition of the derivative, namely $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$, to find the derivative for $f(x) = \frac{3x}{x-1}$.

Derivative: _____

5. (15 points total) For this piecewise function.

$$g(x) = \begin{cases} x^2 + 1 & x < 0 \\ x - 1 & 0 < x \leq 3 \\ 2x - 4 & x > 3 \end{cases}$$

(a) Graph this function.



Now, evaluate the following, or state that the answer does not exist.

(b) $\lim_{x \rightarrow 3} g(x) =$ _____

(c) $\lim_{x \rightarrow 2} g(x) =$ _____

(d) $\lim_{x \rightarrow 0^+} g(x) =$ _____

(e) $\lim_{x \rightarrow 0^-} g(x) =$ _____

(f) $g(\pi) =$ _____

(g) $g(0) =$ _____

6. (16 points total) Find the derivatives of the following functions.

(a) $f(x) = \frac{x^3 + 6x^{-3} + 2}{x^2 + 3x + 1}$

Don't bother to simplify your answer!

$f'(x) =$ _____

(b) $f(x) = (2x^7 + 4x^5 - 6x^{-1})(3x^2 - 7x^{10})$

Don't bother to simplify your answer!

$f'(x) =$ _____

(c) $f(x) = \pi^5 + \pi x^5$

Don't bother to simplify your answer!

$f'(x) =$ _____

Extra Credit: (5 points) True or False.

(a) The derivative of $f(x) = |x|$ is $f'(x) = \frac{x}{|x|}$.

True or False *(circle one)*

(b) Continuity implies differentiability.

True or False *(circle one)*

(c) Differentiability implies continuity.

True or False *(circle one)*

(d) The average change of the function at $x=c$ is given by the slope of the tangent line to the function's curve at $x=c$.

True or False *(circle one)*

(e) The instantaneous change of the function at $x=c$ is given by the slope of the tangent line to the function's curve at $x=c$.

True or False *(circle one)*

Scratch paper:

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