○ : $p$

● : $1-p$

○ : $\overset{\longleftarrow}{1-\alpha}$ $\overset{\longrightarrow}{\alpha}$

● : $\overset{\longleftarrow}{1-\beta}$ $\overset{\longrightarrow}{\beta}$

Q. $X_n \to \infty$?

If only one coin : $\overset{\longleftarrow}{1-\alpha}$ $\overset{\longrightarrow}{\alpha}$ after $n$ tosses $\overset{n(1-\alpha)}{\longleftarrow}$ $\overset{n\alpha}{\longrightarrow}$

Position $X_n \simeq n(2\alpha-1)$ speed $\simeq 2\alpha-1$

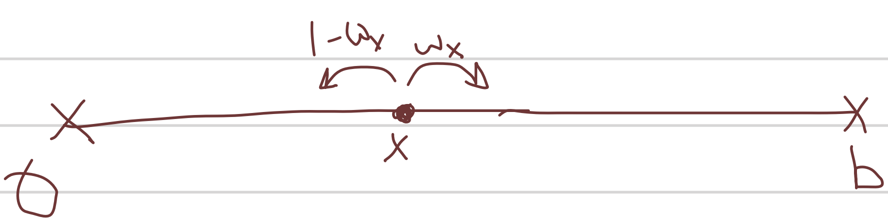
$\alpha > \frac{1}{2} \Rightarrow X_n \to \infty$, $\alpha < \frac{1}{2}$ $X_n \to -\infty$, $\alpha = \frac{1}{2}$ $\underline{\lim} X_n = -\infty$ & $\overline{\lim} X_n = \infty$

Random Env: naive: replace env by $\alpha p + \beta(1-p)$

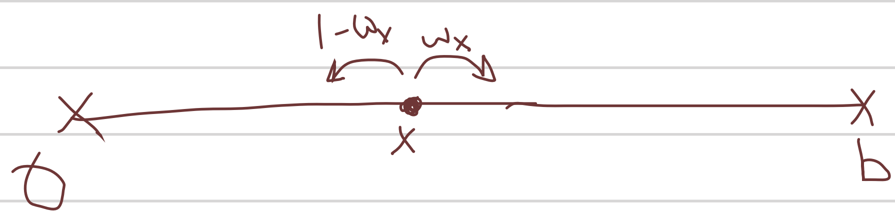
$\alpha p + \beta(1-p) > \frac{1}{2} \Rightarrow X_n \to \infty$ ?

$\alpha p + \beta(1-p) < \frac{1}{2} \Rightarrow X_n \to -\infty$?

$\omega_x = \begin{cases} \alpha & \text{prob } p \\ \beta & \text{prob } 1-p \end{cases}$




$P(\text{start } 1, \text{ hit } b \text{ before hit } 0) = ?$ (then $b \to \infty$ far away)




$w_x = \begin{cases} \alpha & \text{prob } p \\ \beta & \text{prob } 1-p \end{cases}$

$P(\text{start } 1, \text{ hit } b \text{ before hit } 0) = ?$

$f(x) = P(\text{start } x, \text{ hit } b \text{ before } 0)$

$x \in (0,b)$ : $f(x) = w_x f(x+1) + (1-w_x) f(x-1)$ ; $f(0)=0$, $f(b)=1$

$f(x) = w_x f(x) + (1-w_x) f(x)$

$f(x+1) - f(x) = \underbrace{\frac{(1-w_x)}{w_x}}_{\rho_x} \big(f(x) - f(x-1)\big) = \rho_x \rho_{x-1} \cdots \rho_{a+1} f(a+1)$

$f(x) = f(1) + \rho_1 f(1) + \rho_1 \rho_2 f(1) + \cdots + \rho_1 \cdots \rho_{x-1} f(1)$

$x = b$ : $1 = (1 + \rho_1 + \rho_1 \rho_2 + \cdots + \rho_1 \cdots \rho_{b-1}) \underline{\underline{f(1)}}$

$f(1) = \dfrac{1}{1 + \rho_1 + \rho_1\rho_2 + \cdots + \rho_1 \cdots \rho_{b-1}}$

$e^{\log \rho_1 + \cdots + \log \rho_{b-1}} \sim e^{b \overline{\log \rho}}$

P(ever "reach" a far away pt)

$\overline{\log \rho} > 0$ $\left(\log \frac{1-\alpha}{\alpha} p + \log \frac{1-\beta}{\beta}(1-p) > 0\right) \Longrightarrow P(0 \to b) \xrightarrow[b \to \infty]{} 0 \Rightarrow X_n \to -\infty$

$\overline{\log \rho} < 0 \Longrightarrow X_n \to \infty$

$\overline{\log \rho} = 0 \Longrightarrow \underline{\lim} X_n = -\infty$, $\overline{\lim} X_n = \infty$

<u>Ex.</u> $p = 0.7$, $\alpha = 0.7$, $\beta = 0.1 \Longrightarrow \alpha p + \beta(1-p) = 0.63 + 0.03 = 0.66 > 0.5$

$\overline{\log \rho} = 0.7 \log \frac{0.3}{0.7} + 0.3 \log \frac{0.9}{0.1} = 0.0660$ : $X_n \to -\infty$