1a. \(7^3 \cdot 7^2 = 7^{3+2} = 7^5\)

b. \(3^3 \cdot 7^3 = (3 \cdot 7)^3 = 21^3\)

c. \((2^3)^9 = 2^{3 \cdot 9} = 2^{27}\)

d. \(7^3 \div 7^2 = 7^{3-2} = 7\)

e. \(6 \cdot 16 = 2^3 \cdot 2^4 = 2^{3+4} = 2^7\)

f. \(12 \cdot 9 \cdot 2 = 2^3 \cdot 3^2 \cdot 2^1 = 2^{2+1} \cdot 3^2 = 2^3 \cdot 3^2 = (2 \cdot 3)^2 = 6^2\)

1. \((a+b)^n \neq a^n + b^n\). It's easiest to see this by FOILING it out in a simple example. Say, \(n=2\):

\[(a+b)^2 = (a+b)(a+b) = a^2 + ab + ba + b^2\]

Because of these 2 extra terms, they're not equal!

Actually, if \(a\) and \(b\) are whole numbers,

\[(a+b)^n > a^n + b^n\]

(They might be equal if, say, \(b=0\).)
The multiplication algorithm works by converting 13 \& 22 into base 10 blocks, and then filling in a grid of new blocks. Here's 13 \& 22:

Now I'll lay them perpendicular and fill in the space between them. I'll shade the answer.

Totalling it all up, we get 286.
First, try to break it up into 3.

We have □₁ □ left over. First break up the □.

Now try to group again. We can't, so change the 2 longs (□).

Now we have 21 singles (□).

Regroup. Victory! The answer is □₁₁₁₁₁₁₁₁₁₁ or 167.

5. \[23 ÷ 6 = 3 R5.\]

a. Cookie monster needs to eat 6 meals per day, all precisely the same. He has 23 Oreo™ cookies left. How many cookies can he have with each meal if the 23 cookies have to last him the whole day?

The answer (3) is the number of cookies per meal; the remainder is the number he'll have left at the end of the day.

b. Because of his condition, Cookie Monster needs to eat 6 Oreo™ cookies every meal. How many meals will his bag of 23 Oreo™ last him?

The answer is the number of meals he can have; the remainder is the number of extra Oreo™ he'll have left over.
7a. I realize I haven’t told you how to do base 7 division. But the lesson of 43 is:

IT IS THE SAME AS BASE 10 DIVISIONS!

\[
32 \overline{)680_{(7)}}
\]

This is the tricky part.

\[
\begin{align*}
&32 \times 4 = 128_{(7)} \\
&680 - 128 = 552_{(7)} \\
&32 \times 4 = 128_{(7)} \\
&552 - 128 = 424_{(7)} \\
&32 \times 4 = 128_{(7)} \\
&424 - 128 = 296_{(7)} \\
&32 \times 4 = 128_{(7)} \\
&296 - 128 = 168_{(7)} \\
&32 \times 4 = 128_{(7)} \\
&168 - 128 = 40_{(7)} \\
&32 \times 4 = 128_{(7)} \\
&40 - 128 = -88_{(7)} \\
\end{align*}
\]

So \(680_{(7)} \div 32_{(7)} = 21_{(7)} \text{ R } 5_{(7)}\).

b. Ditto multiplication.

\[
\begin{align*}
21 & \text{ \hspace{1cm} } 21 \\
243 & \text{ \hspace{1cm} } 25 \\
1214 & \text{ \hspace{1cm} } 24 \\
+1457 & \text{ \hspace{1cm} } 26 \\
\hline
16304 & \text{ \hspace{1cm} } 50
\end{align*}
\]

Here are the first few base 8 numbers:

<table>
<thead>
<tr>
<th>Base 8 Number</th>
<th>Base 8 Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
</tr>
</tbody>
</table>

Here are the multiplication facts I used (you can figure them out from the left on the left):

\[
\begin{align*}
4 \times 3 &= 14_{(8)}  \\
4 \times 4 &= 20_{(8)}  \\
4 \times 2 &= 10_{(8)}  \\
5 \times 3 &= 17_{(8)}  \\
5 \times 4 &= 16_{(8)}  \\
5 \times 2 &= 12_{(8)}
\end{align*}
\]
8a. The last 2 digits need to be divisible by 4. So the last 2 digits could be any of 80, 84, or 88.
    So 0, 4, or 8.

b. The number must be divisible by 2 and 3. Let's do 3 first.
    To be divisible by 3, we need the digits to sum to something divisible by 3.
    
    \[ 8 + 9 + 7 + 6 + 5 + 0 + 2 + 4 + 3 + 2 + 8 = 54 \]
    This is already divisible by 3, so if we make the last digit 0, 3, 6, or 9, we're set.

    Now let's make sure it's divisible by 2. That means the last digit must be even. So that rules out 3 and 9.

    The options are 0, 4, 6.

9. For one of these divisibility tests to work, the 2 numbers must not have a common factor.
   a. Yes. 4 and 5 have no common factor.
   b. No - 3 and 6 have a common factor.

   Notice that 3 | 12 and 6 | 12, but 18 \( \not| \) 12!
10a. \[ 240 \quad \text{so} \quad 240 = 2^4 \cdot 3 \cdot 5 \]

b. I'm just going to pick 3. Say, \( 24 = 2^3 \cdot 3 \)
\[ 10 = 2 \cdot 5 \]
\[ 6 = 2 \cdot 3 \]

C. 240 has \((4+1) \cdot (1+1) \cdot (1+1) = 20\) factors, because we can have up to 4 2's, 1 3, and 1 5.

d. \[ 600 \quad \text{so} \quad 600 = 2^3 \cdot 3 \cdot 5^2 \]

E. GCF: We need at most 3 2's, one 3, one 5.
\[ \Rightarrow \quad \text{GCF}(240, 600) = 2^3 \cdot 3 \cdot 5 \]
\[ = 120 \]

LCM: We need at least 4 2's, 1 3, 2 5's.
\[ \Rightarrow \quad \text{LCM}(240, 600) = 2^4 \cdot 3^1 \cdot 5^2 = 1200 \]

11a. I'm going to use the Euclidean algorithm, because I'm not crazy.
\[ \text{GCF}(3093, 2359) = \text{GCF}(714, 2359) \]
\[ = \text{GCF}(217, 2359) \]
\[ = \text{GCF}(63, 217) \]
\[ = \text{GCF}(63, 28) \]
\[ = \text{GCF}(7, 28) \]
\[ = \text{GCF}(7, 0) = \left[ 7 \right] \]

b. \[ \frac{2359}{3093} = \frac{337}{439} \]
\[ \frac{337}{439} \div 8093 \neq \frac{2359}{3093} \] I know this is fully simplified, since it is their GREATEST common factor!
12. This is a bit tricky. Notice that if we remove ONE soldier, we can arrange his troops into rows of 3, or rows of 5, the smallest number you can do that with is the LCM, which is 15. So if you remove ONE soldier you have 15. That means we have 16 + 1 = \boxed{16 \text{ soldiers}}.

(or anyway, that’s the smallest possible answer.)

13. Since \(11^2 = 121\) and \(13^2 = 169\), we know from the prime factor test that we only need to test 2, 3, 5, 7, and 11.

Let’s do it.

2: 169 isn’t even, so no.
3: 169 + 9 = 16, not divisible by 3.
5: Last digit isn’t 0 or 5.
7: \[ \begin{array}{r}
7 & \longdiv{169} \\
14 & \\
\hline
29 \\
29 \\
\hline
1
\end{array} \]

11: 169 - 6 = 163, not divisible by 11.

So yes, 139 IS prime.

14a.

\[ \begin{align*}
& (a) \quad \frac{1}{6} \\
& \quad \frac{2}{7} \\
& (c) \\
& (a)
\end{align*} \]

a. well, it’s on one of my tick marks. More to the point, I divided my 3rd mark into 3 pieces, then marked the end of the

b. Since 5 > 4, \( \frac{1}{5} < \frac{1}{4} \).

c. Since \( \frac{7}{8} \), \( \frac{4}{9} > \frac{3}{8} = \frac{3}{8} \). But is it less than \( \frac{3}{4} \)?

\[ \begin{align*}
& \frac{3}{4} > \frac{4}{4} \\
& 7.3 > 4.4 \\
& 21 > 16 \quad \text{so} \quad \frac{3}{4} > \frac{4}{4}.
\end{align*} \]

d. \( \frac{6}{8} = \frac{3}{4} \)
15. We can do it as follows:

\[
\begin{align*}
\frac{1}{2} & < \frac{1 + \frac{4}{2} + \frac{1}{7}}{} < \frac{4}{7} \\
\frac{1}{2} & < \frac{5}{9} < \frac{4}{7}
\end{align*}
\]

That's it!

16. \(\frac{1}{4}\) more than 12 is \(12 + \frac{1}{4} \cdot 12 = 12 + \frac{12}{4} = 12 + \frac{3}{4} = 12 + \frac{3}{4} = 15\)

Indeed, he has \(\frac{3}{4}\) more.

What does less mean? I'll say I have \(\frac{1}{8}\) less than Sam.

\(15 - \frac{1}{8} \cdot 15 = 15 - \frac{15}{8} = 15 - \frac{15}{8} = 15 - \frac{15}{8} = 12\)

Indeed.

How did I know?

\(\frac{12}{15} = \frac{4}{5}\) and \(\frac{4}{5}\) of Sam's candy is \(\frac{1}{5}\) less than \(\frac{5}{8}\) (ie all of it).

17. \(\frac{1}{2}\) is. It corresponds to dividing some thing into 2 parts

and taking none.

\(\frac{2}{6}\) and \(\frac{0}{6}\) aren't. You can't divide something into 0 parts!

18. \(3\frac{1}{6}\) is

\(3\frac{1}{3} + 1\frac{1}{2} = \)

How do we combine these? Divide them into eights.

\(1\frac{1}{2} = \)

Alternatively:

\(3\frac{1}{3} + 1\frac{1}{2} = 3\frac{2}{6} + 1\frac{3}{6} = (3+1) + \frac{2+3}{6} = 4\frac{5}{6}\)
19. \( \frac{3}{2} - \frac{1}{2} \): Let's first cancel 1 whole pie to get:

\[
\begin{array}{c}
\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc - \bigcirc \\
\text{Now I'll cash out a whole pie into halves, to cancel the half.}
\end{array}
\]

\[
\begin{array}{c}
\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc = \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc
\end{array}
\]

And now combine:

\[
\begin{array}{c}
\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc = \bigcirc \bigcirc \bigcirc \bigcirc \bigcirc = \frac{5}{6}
\end{array}
\]

Alternatively:

\[
\frac{3}{2} - \frac{1}{2} = \frac{9 + 1}{3} - \frac{2 + 1}{2} = \frac{10}{3} - \frac{3}{2} = \frac{20}{6} - \frac{9}{6}
\]

\[
= \frac{20 - 9}{6} = \frac{11}{6} = \frac{6 + 5}{6} = \frac{15}{6} = \frac{5}{2}
\]

20. "The rest" is:

\[
\begin{array}{c}
\text{p.b. surprise} + \text{cinn. turtle} + \text{caramel death}
\end{array}
\]

The LCM of 3, 2, 4 is 12.

Counting up, we have 17 twelfths.

So \( \frac{5}{12} \) of the layers are strawberry heaven.

21. I'll let you sort out the details. It would be good to do it w/ diagrams. Here's the answer:

We eat \( 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{7}{4} \) cups per day.

\[36 \div \frac{7}{4} = \frac{36 \cdot 4}{7} = \frac{144}{7} = 5 \cdot \frac{4}{7} = 5 \frac{4}{7} = 5 \frac{4}{7} = 20 \text{ days.}\]

We haven't done fraction division yet. Do this w/ out using it!