Exam 3 Practice Problems

Math 2210-001: Calculus III – Fall 2009

1. Find the extreme values of $f(x,y) = x^3 + 2y^3$ given the constraint $x^2 + y^2 = 1$. *(Hint: You should find six candidate points.)*

2. Find the point on the sphere of radius 1, centered at the origin, which is closest to the point $(4, 2, 1)$.

3. Find the volume of the region in the first octant bounded by the plane $x + 2y + 3z = 6$.

4. Evaluate the integral

$$\int_1^e \int_0^{\ln x} y \, dy \, dx$$

(Hint: It is entirely possible that this is not integrable in its present form.)

5. Find the area of one leaf of a the three-leafed rose defined by the curve $r = 2\sin 3\theta$.

6. Evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)} \, dy \, dx$$

(Hint: It is entirely possible that this is not integrable in its present form.)

7. Find the surface area of the piece of the sphere $x^2 + y^2 + z^2 = 4$ that is inside the cylinder $x^2 + y^2 = 1$.

8. Find the surface area of the portion of the parabolic cylinder $z = x^2$ that is directly above the triangle with vertices $(0, 0, 0)$, $(1, 0, 0)$, and $(1, 1, 0)$.

9. Evaluate the integral

$$\iiint_D xy \, dV$$

where D is the solid region in the first octant bounded above by the hemisphere $z = \sqrt{4 - x^2 - y^2}$.

10. Find the mass of the cube C with sides of length 2, given that the density at a point is equal to the square of the distance from the origin.

11. Find the center of mass of the tetrahedron in the first octant, bounded by the plane $x + y + z = 1$, if the density at a point is equal to the sum of the point's coordinates.

12. Using an integral in cylindrical coordinates, find the volume of the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$.

13. Using an integral in spherical coordinates, find the volume of the portion of the sphere $x^2 + y^2 + z^2 = 4$ above the xy-plane and inside the cone $z^2 = 3(x^2 + y^2)$.

14. Evaluate the integral

$$\iint_R e^{(y-x)^2} \, dA$$

where R is the region bounded by $y = \frac{3}{2}x$, $y = 2x$, and $y = x + 1$. Use the coordinate transformation $x = u + v$, $y = u + 2v$.

15. Evaluate the integral

$$\iint_R 49x^2 y \, dA$$

where R is the region bounded by $2x - y = 1$, $2x - y = -2$, $x + 3y = 0$, $x + 3y = 1$. Use a clever change of variables.