1.a.

$$\vec r(t)=\begin{cases}(\vec v_2-\vec v_1)t+\vec v_1 = 2\vec i\,t+\vec i+2\vec j & t\in[0,1]\\ (\vec v_3-\vec v_2)(t-1)+\vec v_2 = 3\vec j(t-1)+3\vec i+2\vec j & t\in(1,2]\\ (\vec v_1-\vec v_3)(t-2)+\vec v_3 = (-2\vec i-3\vec j)(t-2)+3\vec i+5\vec j & t\in(2,3]\end{cases}$$

b. $$\|\vec v_3-\vec v_2\|+\|\vec v_2-\vec v_1\|+\|\vec v_1-\vec v_3\| = \sqrt{(3-3)^2+(5-2)^2}+\sqrt{(3-1)^2+(2-2)^2}+\sqrt{(1-3)^2+(2-5)^2}$$

$$=\sqrt{3^2}+\sqrt{2^2}+\sqrt{2^2+3^2} = 3+2+\sqrt{13} = \boxed{5+\sqrt{13}}$$

2.a. $\vec r(t)=r\cos t\,\vec i+r\sin t\,\vec j,\quad 0\le t\le 2\pi \quad\Rightarrow\quad \vec r\,'(t)=-r\sin t\,\vec i+r\cos t\,\vec j$

b. $$L=\int_0^{2\pi}\sqrt{(-r\sin t)^2+(r\cos t)^2}\,dt=\int_0^{2\pi}\sqrt{r(\sin^2 t+\cos^2 t)}\,dt=\int_0^{2\pi}\sqrt{r^2}\,dt=\int_0^{2\pi}r\,dt=\boxed{2\pi r}$$

3. $\vec v_1=4\vec i+3\vec j+4\vec k$
$\vec v_2=\vec i-2\vec j+\vec k$
$\vec v_3=-3\vec i+2\vec j+\vec k$

We need 2 vectors in the plane. Let's use

$$\vec v_2-\vec v_1=(1-4)\vec i+(-2-3)\vec j+(1-4)\vec k=-3\vec i+-5\vec j+-3\vec k$$
$$\vec v_3-\vec v_1=(-3-4)\vec i+(2-3)\vec j+(1-4)\vec k=-7\vec i+-1\vec j+-3\vec k$$

So the normal vector to the plane is perpendicular to these:

$$(\vec v_2-\vec v_1)\times(\vec v_3-\vec v_1)=\begin{vmatrix}\vec i & \vec j & \vec k\\ -3 & -5 & -3\\ -7 & -1 & -3\end{vmatrix}=\begin{aligned}&15\vec i+21\vec j+3k\\ &-35k-9\vec j-3\vec i\\ &=12\vec i+12\vec j-32\vec k\end{aligned}$$

The equation for the plane is

$$12x+12y-32z=D$$
$$12(1)+12(-2)-32(1)=D$$
$$-44=D$$

plug in a point, like $(1,-2,1)$

So

$$\boxed{12x+12y-32z=-44}$$

1. Equation of plane is

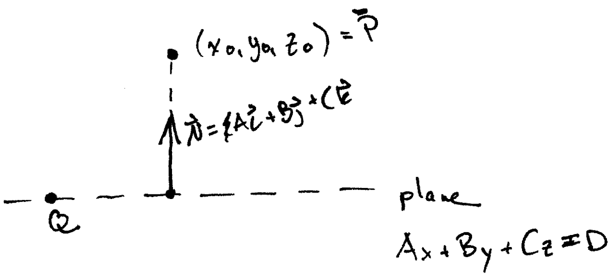
$$2x-y+3z=D$$
$$2(1)-1+3(1)=D$$
$$4=D$$

Plug in $(1,1,1)$

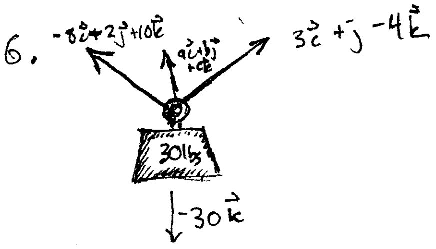
So

$$\boxed{2x-y+3z=4}$$

5.



Hint:
- Find any point on the plane, Q
- Find the vector pointing from Q to P;
- Project it onto the normal vector to the plane $\vec N=A\vec i+B\vec j+C\vec k$. The length of this is what you want.

6. 

So $-8\vec{i}-2\vec{j}+10\vec{k} + 3\vec{i}+\vec{j}-4\vec{k} + a\vec{i}+b\vec{j}+c\vec{k} = d\vec{k}$

$(-8+3+a)\vec{i} + (-2+1+b)\vec{j} + (10-4+c)\vec{k} = d\vec{k}$

so $-8+3+a=0$ $\quad$ $-2+1+b=0$ $\quad$ $10-4+c=d$

$\boxed{a=5}$ $\quad$ $\boxed{b=1}$ $\quad$ $c=d-6$

But, the weight is pulling down with a force of 30lbs. So $d=-(-30)=30$

so $c=30-6=24$.

$\boxed{c=24}$

7. $|\vec{a}\cdot\vec{b}| = \big|\,\|\vec{a}\|\|\vec{b}\|\cos\theta\,\big| = \|\vec{a}\|\|\vec{b}\|\underbrace{|\cos\theta|}_{\cos\theta\,\leq 1} \leq \|\vec{a}\|\|\vec{b}\|$ ∎

8. a. $\vec{u}=4\vec{i}+3\vec{j}$ $\quad$ $\vec{v}=5\vec{i}-12\vec{j}$

$\vec{u}\cdot\vec{v} = 4\cdot5-3\cdot12 = 20-36 = -16$

$\|\vec{u}\| = \sqrt{4^2+3^2} = \sqrt{25} = 5$

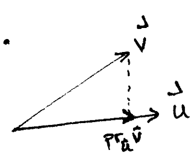
$$\mathrm{pr}_{\vec{u}}\vec{v} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{u}\|^2}\cdot\vec{u} = \frac{-16}{5^2}\cdot(4\vec{i}+3\vec{j})$$
$$= -\frac{16}{25}\cdot(4\vec{i}+3\vec{j})$$
$$= \boxed{\frac{-16\cdot4}{25}\vec{i}+\frac{-16\cdot3}{25}\vec{j}}$$

$$\mathrm{pr}_{\vec{u}}(\mathrm{pr}_{\vec{u}}\vec{v}) = \frac{\vec{u}\cdot\left(\frac{-16\cdot4}{25}\vec{i}+\frac{-16\cdot3}{25}\vec{j}\right)}{\|\vec{u}\|^2}\cdot\vec{u}$$
$$= \frac{-16}{5^2}\cdot(4\vec{i}+3\vec{j})$$
$$= \boxed{\frac{-16\cdot4}{25}\vec{i}+\frac{-16\cdot3}{25}\vec{j}}$$

$\vec{u}\cdot\left(\frac{-16\cdot4}{25}\vec{i}+\frac{-16\cdot3}{25}\vec{j}\right) =$

$= \frac{-16}{25}\cdot(4\vec{i}+3\vec{j})\cdot(4\vec{i}+3\vec{j})$

$= \frac{-16}{25}\cdot(4^2+3^2) = \frac{-16}{25}\cdot25 = -16$

b. 

The projection of $\vec{v}$ onto $\vec{u}$ is parallel to $\vec{u}$; taking its component in the direction of $\vec{u}$ doesn't change anything.

9. Easy! The area of a parallelogram is the magnitude of the cross product of the adjacent sides.

$$\|(3\vec{i}+\vec{j}-4\vec{k})\times(2\vec{i}-2\vec{j})\| = \left\|\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 3 & 1 & -4\\ 2 & -2 & 0\end{vmatrix}\right\| = \left\|(1\cdot0)\vec{i} + -4\cdot2\vec{j} + 3(-2)\vec{k} - 2\cdot1\cdot\vec{k} - 3\cdot0\cdot\vec{j} - (-4)(-2)\vec{i}\right\|$$
$$= \|-8\vec{i}-8\vec{j}-8\vec{k}\|$$
$$= \sqrt{(-8)^2+(-8)^2+(-8)^2} = \sqrt{192}$$
$$= \boxed{8\sqrt{3}}$$

10. $\vec{r}(t) = 2t\vec{i}+t^2\vec{j}+\ln(t)\vec{k}$

$\vec{r}'(t) = 2\vec{i}+2t\vec{j}+\frac{1}{t}\vec{k}$

$\vec{r}''(t) = 2\vec{j}-\frac{1}{t^2}\vec{k}$

a. Position: $\vec{r}(e) = 2e\vec{i}+e^2\vec{j}+\ln(e)\vec{k}$
$= 2e\vec{i}+e^2\vec{j}+1\vec{k}$

Velocity: $\vec{r}'(e) = 2\vec{i}+2e\vec{j}+\frac{1}{e}\vec{k}$

Acceleration: $\vec{r}''(e) = 2\vec{j}+-\frac{1}{e^2}\vec{k}$

b. $$L = \int_1^2\|\vec{r}'(t)\|\,dt = \int_1^2\sqrt{2^2+(2t)^2+\left(\tfrac{1}{t}\right)^2}\,dt = \int_1^2\sqrt{4+4t^2+t^{-2}}\,dt = \int_1^2\sqrt{(2t+1/t)^2}\,dt$$
$$= \int_1^2 2t+1/t\,dt = (t^2+\ln t)\Big|_1^2 = 4+\ln2-1-0 = \boxed{3+\ln 2}$$

1. $\vec{r}(t) = 90\sqrt{2}t\,\vec{i} + 90\sqrt{2}t\,\vec{j} + (64t - 16t^2)\vec{k}$

$\vec{r}'(t) = 90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + (64 - 32t)\vec{k}$

a. position: $\boxed{\vec{r}(0) = 0\vec{i} + 0\vec{j} + 0\vec{k}}$

velocity: $\boxed{\vec{r}'(0) = 90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + 64\vec{k}}$

b. Well, $\vec{r}(4) = 90\sqrt{2}\cdot 4\,\vec{i} + 90\sqrt{2}\cdot 4\,\vec{j} + (64\cdot 4 - 16\cdot 4^2)\vec{k}$

$= 360\sqrt{2}\,\vec{i} + 360\sqrt{2}\,\vec{j} + (256 - 256)\vec{k}$ ← $\vec{k}$ coord. is 0, so it's on the ground.

$= 360\sqrt{2}\,\vec{i} + 360\sqrt{2}\,\vec{j}$

Distance from initial position (found in (a)) is:

$\|\vec{r}(4) - \vec{r}(0)\| = \|(360\sqrt{2} - 0)\vec{i} + (360\sqrt{2} - 0)\vec{j} + (0-0)\vec{k}\| = \sqrt{(360\sqrt{2})^2 + (360\sqrt{2})^2} = 360\sqrt{2}\sqrt{1+1}$

$= 360\sqrt{2}\cdot\sqrt{2} = 360\cdot 2 = \boxed{720}$

c. position @ t=1: $\vec{r}(1) = 90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + (64-16)\vec{k} = 90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + 48\vec{k}$

velocity @ t=1: $\vec{r}'(1) = 90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + (64-32)\vec{k} = 90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + 32\vec{k}$

Tangent line is

$\vec{L}(t) = \vec{r}'(1)t + \vec{r}(1)$

$\boxed{\vec{L}(t) = (90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + 32\vec{k})t + 90\sqrt{2}\,\vec{i} + 90\sqrt{2}\,\vec{j} + 48\vec{k}}$

2. $\vec{r}(t) = 2t\,\vec{i} + t^2\vec{j} + \frac{1}{3}t^3\vec{k}$

$\vec{r}'(t) = 2\vec{i} + 2t\vec{j} + t^2\vec{k}$

$\|\vec{r}'(t)\| = \sqrt{2^2 + (2t)^2 + (t^2)^2} = \sqrt{4 + 4t^2 + t^4} = \sqrt{(t^2+2)^2} = t^2 + 2$

$\vec{T} = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|} = \frac{2\vec{i} + 2t\vec{j} + t^2\vec{k}}{t^2+2} = \frac{2}{t^2+2}\vec{i} + \frac{2t}{t^2+2}\vec{j} + \frac{t^2}{t^2+2}\vec{k}$

$\vec{r}''(t) = 2\vec{j} + 2t\vec{k}$

$a_T = \vec{T}\cdot r'' = \frac{2}{t^2+2}\cdot 0 + \frac{2t}{t^2+2}\cdot 2 + \frac{t^2}{t^2+2}\cdot 2t = \boxed{\frac{4t + 2t^3}{t^2+2} = a_T}$

$a_N = \|\vec{T}\times r''\| = \left\|\frac{1}{t^2+2}(2\vec{i} + 2t\vec{j} + t^2\vec{k})\times(2\vec{j} + 2t\vec{k})\right\| = \left|\frac{1}{t^2+2}\right|\cdot\|(2\vec{i}+2t\vec{j}+t^2\vec{k})\times(2\vec{j}+2t\vec{k})\|$

$= \frac{1}{t^2+2}\left\|\begin{vmatrix} i & j & k \\ 2 & 2t & t^2 \\ 0 & 2 & 2t \end{vmatrix}\right\| = \frac{1}{t^2+2}\|4t^2\vec{i} + t^2\cdot 0\vec{j} + 2\cdot 2\vec{k} - 2t\cdot 0\vec{k} - 2\cdot 2t\vec{j} - 2\cdot t^2\vec{i}\|$

$= \frac{1}{t^2+2}\|2t^2\vec{i} - 4t\vec{j} + 4\vec{k}\| = \frac{1}{t^2+2}\sqrt{4t^4 + 16t^2 + 16} = \frac{2}{t^2+2}\sqrt{(t^2+2)^2} = \boxed{2}$

For efficiency, I've rewritten $\vec{T}$ thus: $\vec{T} = \frac{1}{t^2+2}\cdot(2\vec{i} + 2t\vec{j} + t^2\vec{k})$

13a. $\vec{r}(t) = t\cos(t)\vec{i} + t\sin(t)\vec{j}$

$x = t\cos t$

$x' = \cos t - t\sin t$

$x'' = -\sin t - \sin t - t\cos t$

$= -2\sin t - t\cos t$

$y = t\sin t$

$y' = \sin t + t\cos t$

$y'' = \cos t + \cos t + -t\sin t$

$= 2\cos t - t\sin t$

$K = \frac{|x'y'' - y'x''|}{((x')^2 + (y')^2)^{3/2}}$

$= \frac{|(\cos t - t\sin t)(2\cos t - t\sin t) - (\sin t + t\cos t)(-2\sin t - t\cos t)|}{((\cos t - t\sin t)^2 + (\sin t + t\cos t)^2)^{3/2}}$

$= \frac{|2\cos^2 t - \cancel{t\sin t\cos t} - 2t\sin t\cos t + t^2\sin^2 t + 2\sin^2 t + t\sin t\cos t + 2t\sin t\cos t + t^2\cos^2 t|}{(\cos^2 t - 2t\cos t\sin t + t^2\sin^2 t + \sin^2 t + t\sin t\cos t + t^2\cos^2 t)^{3/2}}$

$= \frac{|2(\cos^2 t + \sin^2 t) + t^2(\cos^2 t + \sin^2 t)|}{((\cos^2 t + \sin^2 t) + t^2(\sin^2 t + \cos^2 t))^{3/2}} = \boxed{\frac{2+t^2}{(1+t^2)^{3/2}}}$

b. same as $y = \frac{1}{3}x^3$;

$y' = x^2$

$y'' = 2x$

$K = \frac{|y''|}{(1+(y')^2)^{3/2}} = \frac{|2x|}{(1+(x^2)^2)^{3/2}} = \boxed{\frac{|2x|}{(1+x^4)^{3/2}}}$

4

$$\kappa = \frac{|y''|}{|1+(y')^2|^{3/2}} = \frac{|e^x|}{|1+(e^x)^2|^{3/2}}$$

$$= \frac{e^x}{(1+e^{2x})^{3/2}}$$

$y = e^x$
$y' = e^x$
$y'' = e^x$

$$\kappa'(x) = \frac{(1+e^{2x})^{3/2}\cdot e^x - e^x \cdot \frac{3}{2}(1+e^{2x})^{1/2}\cdot e^{2x}}{(1+e^{2x})^3}$$

$$= \frac{(1+e^{2x})e^x - \frac{3}{2}e^{3x}}{(1+e^{2x})^{5/2}}$$

$$= \frac{e^x + e^{3x} - \frac{3}{2}e^{3x}}{(1+e^{2x})^{5/2}} = \frac{e^x - \frac{1}{2}e^{3x}}{(1+e^{2x})^{5/2}}$$

$$= \frac{e^x(1-\frac{1}{2}e^{2x})}{(1+e^{2x})^{5/2}}$$

To find maximum curvature, set $\kappa'(x) = 0$:

$\kappa'(x) = 0$ when the numerator is equal to 0.

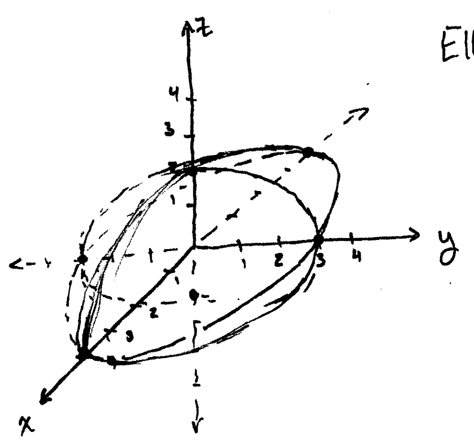
$$e^x(1-\tfrac{1}{2}e^{2x}) = 0$$

$\cancel{e^x = 0}$ or $1 - \frac{1}{2}e^{2x} = 0$

$$\tfrac{1}{2}e^{2x} = 1$$
$$e^{2x} = 2$$
$$2x = \ln 2$$
$$\boxed{x = \frac{\ln 2}{2}}$$

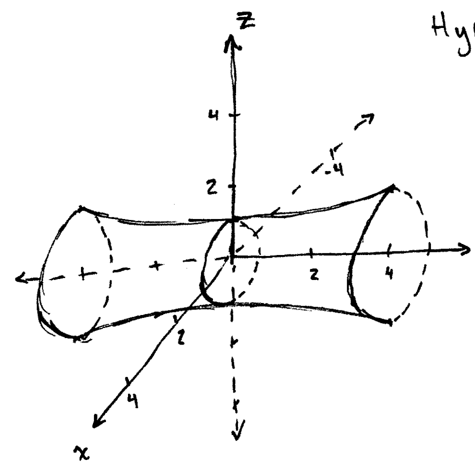
It's a good idea to make sure this is a maximum. I'll let you do that.

5 a.



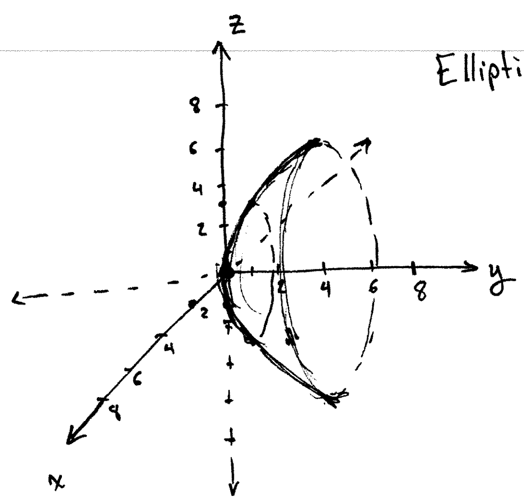
Ellipsoid.

b.



Hyperboloid of 1 sheet.

c.



Elliptic Paraboloid

I haven't shown much work on these.
You should show more!