Exam 1 Practice Problems

Math 2210-001: Calculus III – Fall 2009

1. Given the triangle with vertices $\mathbf{v}_1 = \mathbf{i} + 2\mathbf{j}$, $\mathbf{v}_2 = 3\mathbf{i} + 2\mathbf{j}$, and $\mathbf{v}_3 = 3\mathbf{i} + 5\mathbf{j}$:
   (a) Write down a piecewise, vector-valued function describing a path along the triangle. It should go from $\mathbf{v}_1$ through $\mathbf{v}_2$ and $\mathbf{v}_3$, and then back to $\mathbf{v}_1$.
   (b) Find the perimeter of the triangle.
2. Given a circle of radius r centered at the origin:
   (a) Write down a vector-valued function describing a path around the circle.
   (b) Use the formula for the length of a curve to show that the perimeter of a circle is equal to $2\pi r$.
3. Find the equation for the plane through the points $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, $\mathbf{i}-2\mathbf{j}+\mathbf{k}$, and $-3\mathbf{i}+2\mathbf{j}+\mathbf{k}$.
4. Find the equation for the plane parallel to the plane defined by $2x - y + 3z = 1$, but passing through the point $(1, 1, 1)$.
5. **(Hard!)** Show that the distance from the point (x_0, y_0, z_0) to the plane $Ax+By+Cz = D$ is given by the formula
$$L = \frac{|Ax_0 + By_0 + Cz_0 - D|}{\sqrt{A^2 + B^2 + C^2}}$$
6. A weight of 30 lbs is suspended by three wires with resulting tensions $3\mathbf{i} + 4\mathbf{j} + 15\mathbf{k}$, $-8\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}$, and $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$. Find a, b, and c, assuming $\mathbf{k}$ points straight up.
7. **(Hard!)** Prove that
$$|\mathbf{a} \cdot \mathbf{b}| \leq \|\mathbf{a}\|\|\mathbf{b}\|$$
8. Given $\mathbf{u} = 4\mathbf{i} + 3\mathbf{j}$ and $\mathbf{v} = 5\mathbf{i} - 12\mathbf{j}$:
   (a) Calculate $\text{pr}_\mathbf{u}\,\mathbf{v}$ and $\text{pr}_\mathbf{u}\,(\text{pr}_\mathbf{u}\,\mathbf{v})$
   (b) Explain, with a diagram or a few sentences, why the above two answers are equal.
9. Find the area of the parallelogram with adjacent sides given by $3\mathbf{i}+\mathbf{j}-4\mathbf{k}$ and $2\mathbf{i}-2\mathbf{j}$.
10. Suppose an object has the position function
$$\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \ln(t)\mathbf{k}$$
   (a) Find the position, velocity, and acceleration of the object at time $t = e$.
   (b) Find the distance the object has travelled from time $t = 1$ to time $t = 2$.

11. At the 1908 US Open, Fred McLeod struck a golf ball at time $t = 0$. Its position function following that was given by the function

$$\mathbf{r}(t) = 90\sqrt{2}t\mathbf{i} + 90\sqrt{2}t\mathbf{j} + (64t - 16t^2)\mathbf{k}$$

(a) Find the initial position and initial velocity of the golf ball (assuming distances in feet and time in seconds).

(b) Show that the golf ball strikes the ground at time $t = 4$, and determine the distance from its initial position.

(c) Suppose Fred McLeod's arch-enemy Nikola Tesla, looking to spoil the golf game, activated his antigravity device at time $t = 1$. Find the equation for the line that the golf ball will travel along.

12. Find the tangential and normal compoents of acceleration for the curve given by

$$\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} + \frac{1}{3}t^3\mathbf{k}$$

13. Find the curvature for each of the following curves:

(a) $\mathbf{r}(t) = t\cos(t)\mathbf{i} + t\sin(t)\mathbf{j}$

(b) $\mathbf{r}(t) = t\mathbf{i} + \frac{1}{3}t^3\mathbf{k}$

14. Find the point where the curvature of $y = e^x$ is at its maximum.

15. Graph the following surfaces:

(a) $\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} = 1$

(b) $9x^2 - y^2 + 9z^2 = 9$

(c) $9x^2 + 4z^2 - 36y = 0$