Name: Due Friday, November 7

Worksheet 3

1. Evaluate the following **without using a calculator**.

   (a) $\log(10000) =$ $\log(10^4) = 4\log(10) = 4 \cdot 1 = 4$

   (b) $\log(0.0001) =$ $\log(10^{-4}) = -4\log(10) = -4 \cdot 1 = -4$

2. Solve for x:

   (a) $3^x = 243$

   $\log(3^x) = \log(243)$

   $x\log(3) = \log(243)$

   $x = \dfrac{\log(243)}{\log(3)}$

   $\boxed{x = 5}$

   (b) $1000 = 500 \cdot \left(1 + \frac{0.03}{12}\right)^{12 \cdot x}$

   $\div 500$

   $2 = \left(1 + \frac{0.03}{12}\right)^{12x}$

   $\log(2) = \log\left(\left(1 + \frac{0.03}{12}\right)^{12x}\right)$

   $\log(2) = 12x\log\left(1 + \frac{0.03}{12}\right)$

   $\dfrac{\log(2)}{\log\left(1 + \frac{0.03}{12}\right)} = x$

   $\dfrac{\log(2)}{12\log\left(1 + \frac{0.03}{12}\right)} = x$

   $\boxed{x = 23.134}$

3. Jane owns a pastry shop in Oakland. She's sick of making her own tortes, so she bakes a pastry that can reproduce by parthenogenesis. She has 40 pastries today, and their population is increasing at a rate of 10% per hour. When she has 1,000,000 pastries, they will be able to take over the city.

   (a) How long does it take the pastry population to double?

   $2 = 1 \cdot (1 + 0.10)^t$

   $\log(2) = \log((1 + 0.10)^t)$

   $\log(2) = t\log(1 + 0.1)$

   $\dfrac{\log(2)}{\log(1 + 0.1)} = t$

   $\boxed{t = 7.2725 \text{ hours}}$

   (b) How long does it take the pastries to take over the city?

   $1{,}000{,}000 = 40 \cdot (1 + 0.10)^t$

   $\div 40$

   $25{,}000 = (1 + 0.10)^t$

   $\log(25{,}000) = \log((1 + 0.10)^t)$

   $\log(25{,}000) = t\log(1 + 0.10)$

   $\dfrac{\log(25{,}000)}{\log(1 + 0.10)} = t$

   $\boxed{t = 106.24 \text{ hours}}$

4. To build her mutant pastries, Jane had to invest in a lump of radioactive Californium (Cf-252). One year ago, she bought 500 grams of Cf-252. Now she has 385 grams. What is the half-life of Cf-252?

$$500 = 385 \cdot 2^{-1/T_{half}}$$

$\div 385$

$$\frac{500}{385} = 2^{-1/T}$$

$$\log\left(\frac{500}{385}\right) = \log\left(2^{-1/T}\right)$$

$$\log\left(\frac{500}{385}\right) = \frac{-1}{T}\log(2)$$

$*T \quad *T$

$$T\log\left(\frac{500}{385}\right) = -\log(2)$$

$$T = -\frac{\log(2)}{\log\left(\frac{500}{385}\right)} = \boxed{2.652 \text{ years}}$$

5. To deal with the pastry problem, the government of Oakland plans to commission my friend Eugene to genetically engineer giant pastry-eating mice. Suppose they buy Q_0 mice from Eugene, and the mouse population grows at a rate of P percent per hour. How long does it take their population to double? (Once you're finished with this problem, you'll have a formula for the doubling time! If you replace $\frac{P}{100}$ with r, you'll get the same formula as in the book.) The set-up would be

$$2Q_0 = Q_0 \cdot \left(1+\tfrac{P}{100}\right)^t$$

& then solve for t.

Instead, I'll solve for t in

$$2Q_0 = Q_0 \cdot (1+r)^t$$

$\div Q_0$

$$2 = (1+r)^t$$

$$\log(2) = \log\left((1+r)^t\right)$$

$$\log(2) = t\log(1+r)$$

$\div \log(1+r)$

$$\boxed{\frac{\log(2)}{\log(1+r)} = t}$$