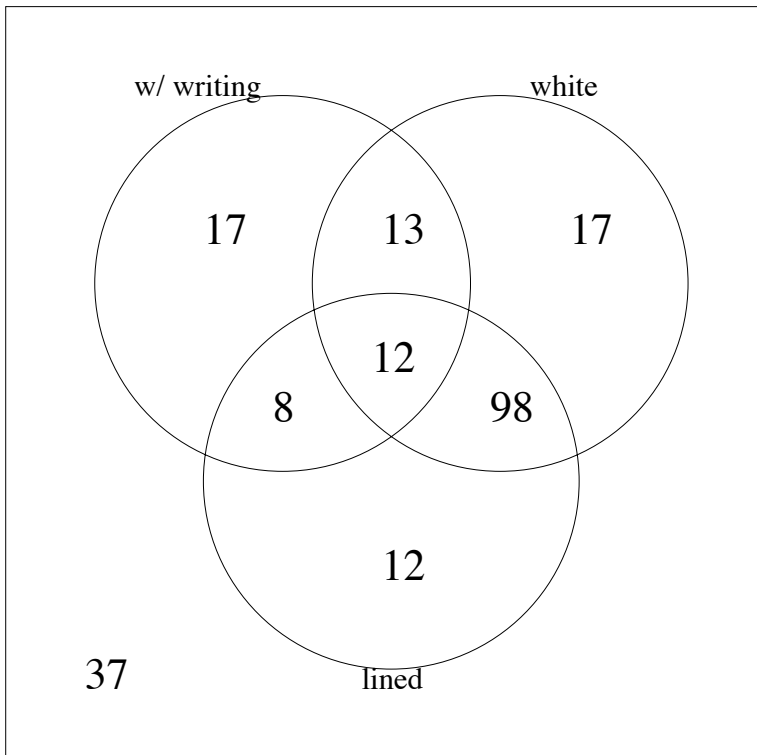


Review for Exam #1 Solutions

1. Betty looks through her stack of scratch paper and records the following information regarding the scratch paper.

- 214 sheets of paper.
- 130 of them are lined
- 140 of them are white
- 50 of them have some writing on them
- 110 of the lined paper are white
- 12 of the lined white paper have writing on them
- 25 of the white paper have writing on them
- 20 of the lined paper have writing on them

Draw a Venn diagram to describe this information and use it to determine how many sheets of paper Betty has that are either white or lined (this includes those that are both).

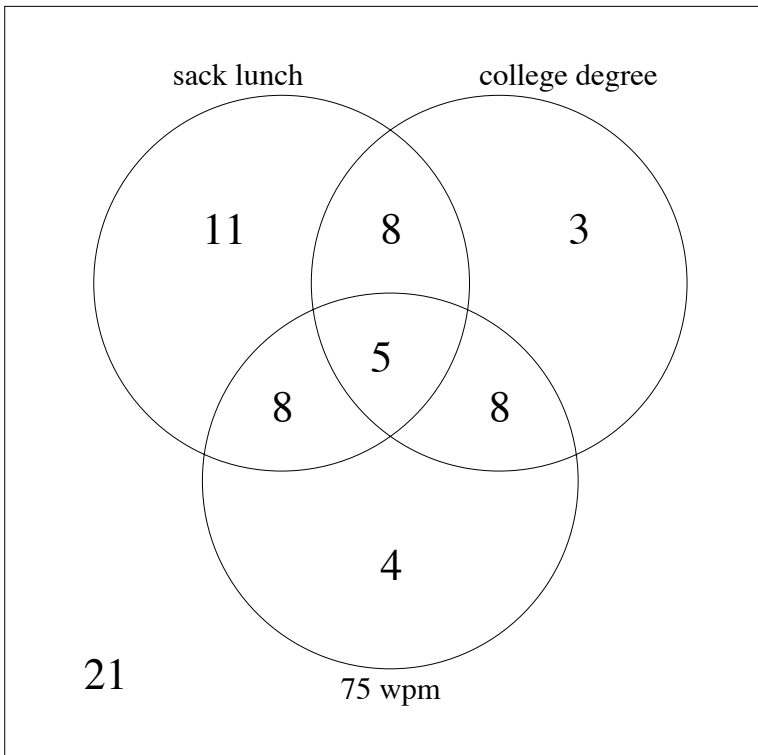


So there are $214 - 37 - 17 = 160$ sheets of paper that are white or lined.

2. Steve Keyboard works in an office building. He finds out the following information regarding the 68 people on his floor.

- 25 can type over 75 words per minute
- 24 have a college degree
- 32 bring sack lunches
- 13 of the college graduates bring sack lunches
- 13 of the college graduates can type over 75 words per minute
- 13 of those that type over 75 words per minute bring sack lunches
- 5 of the college graduates that bring sack lunches can type over 75 words per minute

Draw a Venn diagram to describe this information and use it to determine how many people there are in the office who do not bring sack lunches, do not type over 75 words per minute, but do have a college degree.



So there are 3 college graduates who do not bring sack lunches and do not type 75 wpm.

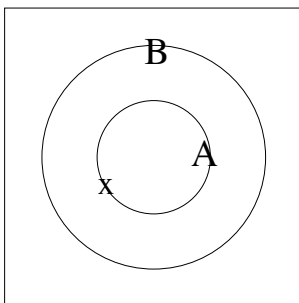
3. Determine whether the following deductive argument is valid or invalid. If it is valid, determine whether or not the argument is sound.

Premise #1: All dwarf planets have mass less than one hundredth times the mass of the earth

Premise #2: The mass of Ceres is less than one hundredth times the mass of the earth

Conclusion: Ceres is a dwarf planet

Let A be the set of all dwarf planets and let B be the set of all objects whose mass is less than 1/100th times the mass of the Earth. Then A is a subset of B by the first premise and the diagram for the argument is:



So the argument is invalid because it cannot be determined whether Ceres is in A or not.

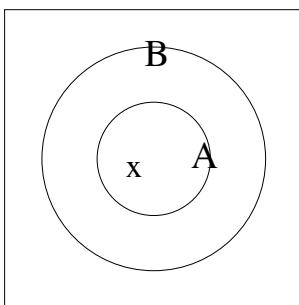
4. Determine whether the following deductive argument is valid or invalid. If it is valid, determine whether or not the argument is sound.

Premise #1: Every region with pyramid structures was home to an ancient civilization

Premise #2: Cydonia is a region of Mars with pyramid structures

Conclusion: Cydonia was home to an ancient civilization

Let A be the set of regions with pyramid structures and let B be the set of regions that were home to an ancient civilization. Then A is a subset of B by the first premise and the diagram for the argument is:



So the argument is valid since Cydonia is necessarily in B . It is not sound, since the first premise probably is not true.

5. Suppose Maxine's car gets 35 miles per gallon. What is her fuel efficiency in feet per fluid ounce?
 (128 fluid ounces = 1 gallon, 1 mile = 5280 feet)

$$\left(35 \frac{\text{mi}}{\text{gal}} \right) \left(\frac{1 \text{ gal}}{128 \text{ oz}} \right) \left(\frac{5280 \text{ ft}}{1 \text{ mi}} \right) = 1,443.75 \frac{\text{ft}}{\text{oz}}$$

6. It is estimated that when the natural dam at Red Rock Pass broke, water from Lake Bonneville at a rate of 15,000,000 cubic feet per second. Compute the flow rate of the water in cubic miles per day.

(1 mile = 5280 feet, 1 hour = 3600 seconds, 1 day = 24 hours)

$$\left(15,000,000 \frac{\text{ft}^3}{\text{sec}} \right) \left(\frac{1 \text{ mi}}{5280 \text{ ft}} \right) \left(\frac{1 \text{ mi}}{5280 \text{ ft}} \right) \left(\frac{1 \text{ mi}}{5280 \text{ ft}} \right) \left(\frac{3600 \text{ sec}}{1 \text{ hr}} \right) \left(\frac{24 \text{ hr}}{1 \text{ day}} \right) = 8.804 \frac{\text{mi}^3}{\text{day}}$$

7. Outer space sales tax just increased to 3.8%. If Petra's astronaut ice cream cost \$12.44 **after** outer space tax, how much did her ice cream cost before tax?

So 12.44 is 3.8% more than the price before tax. If x is the price before tax, then 12.44 is 103.8% of x ; that is,

$$12.44 = 1.038x$$

$$\frac{12.44}{1.038} = x$$

$$11.98 = x$$

So the price before tax was \$11.98.

8. Last year Jupiter Space Suits cost \$1,485. This year Jupiter Space Suits cost \$1,678. By what percent did the cost of Jupiter Space Suits increase from last year to this year?

We want to know: 1,678 is what percent more than 1,485.

$$1,678 = x(1,485)$$

$$1.1299 = x$$

So 1,678 is 112.99% of 1,485, meaning that 1,678 is 12.99% more than 1,485; that is, the cost of the suits increased by 12.99%.

9. The outer space economy is very unpredictable. Two years ago, outer space inflation caused the value of outer space currency to increase by 2.1%. Last year, the value of outer space currency increased by 6.3%. This year, the value of outer space currency decreased by 8.8% due to the banking collapse. Find the total percentage increase or decrease over the last two years.

A 2.1% increase corresponds to multiplying by 1.021. A 6.3% increase corresponds to multiplying by 1.063. A 8.8% decrease corresponds to multiplying by .912. So the total change corresponds to multiplying by

$$(1.021)(1.063)(.912) = .9898$$

so the total change is a net decrease by 1.02%.

10. Laura just sold a really fancy sweater she knitted out of vicuña wool she spun herself from her own vicuña farm in Peru. She made \$3000. Now she wants to deposit it in a bank account. Chevy Chase Bank is offering 4.11% APR compounded monthly, and the Banco de Crédito del Peru is offering 4.1% APR, compounded continuously.

- (a) If she puts her money with Chevy Chase Bank, how much will she have in 10 years?

$$A = 3,000 \left(1 + \frac{.0411}{12} \right)^{(12)(10)} = 4,521.80$$

- (b) If she puts her money with Banco de Crédito del Peru, how much will she have in 10 years?

$$A = 3,000e^{(.041)(10)} = 4,520.45$$

- (c) Calculate the APY for each account. Which is the better account?

For the Chevy Chase Bank account,

$$100 \left(1 + \frac{.0411}{12} \right)^{12} = 104.1883$$

so the APY is 4.1883%.

For the Banco de Crédito del Peru account,

$$100e^{.041} = 104.1852$$

so the APY is 4.1852%.

The Chevy Chase Bank account is better.

- (d) Suppose she puts her money in the better account. In 10 years, what percentage of her balance will be from interest?

In the better account, Laura ends up with \$4,521.80. \$3,000 of it was deposited, so \$1,521.80 came from interest. We want to know: 1,521.80 is what percent of 4,521.80.

$$\begin{aligned} 1521.80 &= x(4521.80) \\ .3365 &= x \end{aligned}$$

So 1,521.80 is 33.65% of 4,521.80.

11. Arjen wants to have \$40,000 21 years from now. His savings account has an APR of 5.3%, compounded daily.

- (a) How much should he deposit now in order to reach his goal?

$$\begin{aligned} 40,000 &= P \left(1 + \frac{.053}{365} \right)^{(365)(21)} \\ 40,000 &= P(3.043229017) \\ 13,143.93 &= P \end{aligned}$$

So Arjen needs to deposit \$13,143.93 in the bank right now.

- (b) By what percent did his deposit increase in the 21 years?

He deposits \$13,143.93 and ends up with \$40,000. We want to know: 40,000 is what percent more than 13,143.93.

$$\begin{aligned} 40,000 &= x(13,143.93) \\ 3.04322 &= x \end{aligned}$$

So 40,000 is 304.322% of 13,143.93; that is, 40,000 is 204.322% of 13,143.93.

12. Use scientific notation to do the following computations.

(a)

$$(87,100,000,000,000,000,000)(.000000000000000011) = (8.71 \times 10^{19})(1.1 \times 10^{-16}) = 9.581 \times 10^3$$

(b)

$$\frac{.00000000000000000626}{.00000000000000313} = \frac{6.26 \times 10^{-19}}{3.13 \times 10^{-14}} = 2 \times 10^{-19+14} = 2 \times 10^{-5}$$

(c)

$$\frac{440,000,000,000,000}{.0000000000000004} = \frac{4.4 \times 10^{14}}{4 \times 10^{-17}} = 1.1 \times 10^{14+17} = 1.1 \times 10^{31}$$

(d)

$$\frac{.000000000000012}{30,000,000,000,000,000} = \frac{1.2 \times 10^{-13}}{3 \times 10^{16}} = .4 \times 10^{-13-16} = .4 \times 10^{-29} = 4 \times 10^{-30}$$

13. At some point in the future, Laura would like to expand into farming cashmere goats, so every quarter she is depositing \$2500 of her profits into a goat account at 4.5% APR. How much will she have in 10 years?

$$\begin{aligned} A &= 2500 \times \frac{\left[\left(1 + \frac{0.045}{4}\right)^{(4 \cdot 10)} - 1 \right]}{\left(\frac{0.045}{4}\right)} \\ &= \$125,417.08 \end{aligned}$$

14. Laura is looking to expand her vicuña farm. She is making about \$1000 profit per month. She decides to take out a 20 year loan from Banco de Crédito del Peru at 5% APR. If she spends all of her profit, every month, on paying down her loan, how large a loan can she afford?

$$1000 = \frac{P \times \left(\frac{0.05}{12}\right)}{\left[1 - \left(1 + \frac{0.05}{12}\right)^{(-12 \cdot 20)}\right]} \quad (1)$$

$$\frac{1000}{\left(\frac{\left(\frac{0.05}{12}\right)}{\left[1 - \left(1 + \frac{0.05}{12}\right)^{(-12 \cdot 20)}\right]}\right)} = P \quad (2)$$

$$P = \$151,525.31 \quad (3)$$