$\boxed{HW\ 7}$

1a. $\log(10) = 1$ since $10^1 = 10$

b. $\log(1) = \log(10^0) = 0\log(10) = 0$

c. $\log(10^4) = 4\log(10) = 4$

d. $\log(100{,}000{,}000) = \log(10^8) = 8\log(10) = 8$

e. $\log(10^{-3}) = -3\log(10) = -3$

f. $\log(0.001) = \log(10^{-3}) = -3\log(10) = -3$

2a. $\log(4^x) = 7$

$x\log(4) = 7$

$x = \dfrac{7}{\log(4)}$

$= \boxed{11.627}$

b. $3 \cdot 2^x = 384$

$\div 3$

$2^x = 128$

$\log(2^x) = \log(128)$

$x\log(2) = \log(128)$

$x = \dfrac{\log(128)}{\log(2)}$

$= \boxed{7}$

c. $e^x + 4 = 10$

$-4 \quad -4$

$e^x = 6$

$\log(e^x) = \log(6)$

$x\log(e) = \log(6)$

$\div \log(e)$

$x = \dfrac{\log(6)}{\log(e)}$

$= \boxed{1.792}$

d. $1000 \cdot \left(1 + \frac{0.04}{12}\right)^x = 2000$

$\div 1000$

$\left(1 + \frac{0.04}{12}\right)^x = 2$

$\log\left(\left(1 + \frac{0.04}{12}\right)^x\right) = \log(2)$

$x\log\left(1 + \frac{0.04}{12}\right) = \log(2)$

$x = \dfrac{\log(2)}{\log\left(1 + \frac{0.04}{12}\right)}$

$= \boxed{208.291}$

3.

$$A(t) = P \cdot \left(1 + \frac{APR}{n}\right)^{n \cdot y}$$

$$110{,}000 = 70{,}000 \cdot \left(1 + \frac{0.07}{12}\right)^{12 \cdot y}$$

$$\div 70{,}000$$

$$\frac{110{,}000}{70{,}000} = \left(1 + \frac{0.07}{12}\right)^{12y}$$

$$\log\left(\frac{110000}{70000}\right) = \log\left(\left(1 + \frac{0.07}{12}\right)^{12 \cdot y}\right)$$

$$\log\left(\frac{110000}{70{,}000}\right) = 12y \ \log\left(1 + \frac{0.07}{12}\right)$$

$$\div \log\left(1 + \frac{0.07}{12}\right)$$

$$\frac{\log\left(\frac{110{,}000}{70{,}000}\right)}{\log\left(1 + \frac{0.07}{12}\right)} = 12y$$

$$\div 12$$

$$\frac{\log\left(\frac{110{,}000}{70{,}000}\right)}{12 \cdot \log\left(1 + \frac{0.07}{12}\right)} = y$$

$y = 6.476$ years

$6.476 \text{ years} \cdot \frac{12 \text{ mo}}{1 \text{ year}} = 5.709$ months

So $\boxed{\text{6 years \& 6 months.}}$

4a. $Q(t) = Q_0 \cdot (1 + r)^t$

$$2 = 1 \cdot (1 + 0.07)^t$$

$$\log(2) = \log\left((1 + 0.07)^t\right)$$

$$\log(2) = t \log(1 + 0.07)$$

$$t = \frac{\log(2)}{\log(1.07)} = \boxed{10.245 \text{ weeks}}$$

b. $5 \text{ months} \cdot \frac{4 \text{ weeks}}{1 \text{ month}} = 20$ weeks

Method 1:

$$Q(t) = 1000 \cdot (1 + 0.07)^t$$

$$Q(20) = 1000 \cdot (1 + 0.07)^{20} = 3869.68$$

so $\boxed{3869 \text{ jellyfish}}$

Method 2:

$$Q(t) = 1000 \cdot 2^{t/10.245}$$

$$Q(20) = 1000 \cdot 2^{20/10.245} = 3869.68$$

so $\boxed{3869 \text{ jellyfish}}$

c. $1{,}000{,}000 = 1{,}000 \cdot (1 + 0.07)^t$

$$\div 1000$$

$$1000 = (1 + 0.07)^t$$

$$\log(1000) = \log\left((1 + 0.07)^t\right)$$

$$3 = t \ \log(1 + 0.07)$$

$$t = \frac{3}{\log(1.07)} = \boxed{102.097 \text{ weeks}}$$