Name: Due October 24th

Homework 6

1. My friend Jenny just sold her motorcycle. She got \$2000 for it. She's going to put it in an account, and once she has \$3000 she's going to go on vacation.

   (a) Suppose she puts it in a bank account that makes 5% APR, compounded monthly. Will she have enough in five years?

   $$\begin{aligned} A &= \$2000\left(1+\frac{0.05}{12}\right)^{12\cdot 5} \\ &= \$2566.71 \end{aligned}$$

   No, she will not have enough.

   (b) Suppose she puts it in a mutual fund that makes 8% APR, compounded continuously. Will she have enough in five years?

   $$\begin{aligned} A &= \$2000e^{0.08\cdot 5} \\ &= \$2983.64 \end{aligned}$$

   No, she will not have enough.

2. Now, some practice with functions. Tell me what the dependant and independent variables are for each of the following functions.

   (a) The water pressure at a given depth.

   Independent variable: Depth
   Dependent variable: Water pressure

   (b) How light it is out at a given time.

   Independent variable: Time of day
   Dependent variable: How light it is

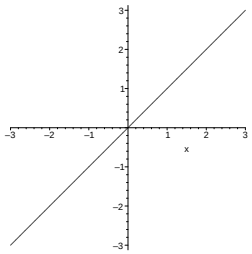
   (c) The amount of gas you have in your car after you've driven a given distance.

   Independent variable: Distance you've driven
   Dependent variable: Amount of gas in your car

3. For each of the following functions:

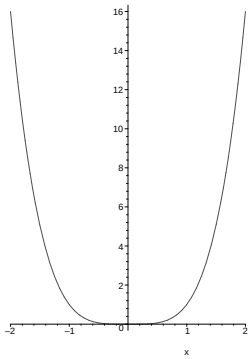
   - Tell me the domain and range
   - Evaluate $f(5)$

(a) $f(x) = x$



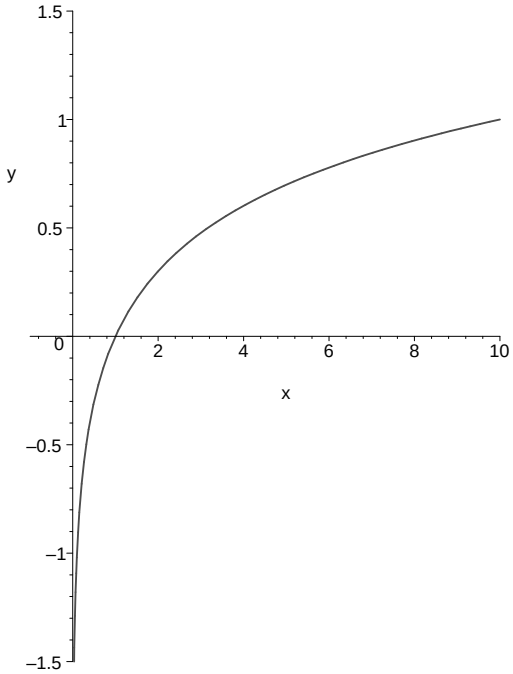
Domain: $(-\infty, \infty)$, or "all real numbers"
Range: $(-\infty, \infty)$, or "all real numbers"
$f(5) = 5$

(b) $f(x) = x^4$



Domain: $(-\infty, \infty)$, or "all real numbers"
Range: $[0, \infty)$, or "all non-negative numbers" (that is, positive numbers and 0)
$f(5) = 625$

(c) $f(x) = \log(x)$ You'll certainly need your calculator for this one. Let me know if you need help.



Domain: $(0, \infty)$, or "all positive numbers"
Range: $(-\infty, \infty)$, or "all real numbers" (that is, positive numbers and 0)
$f(5) =$ 0.698970004

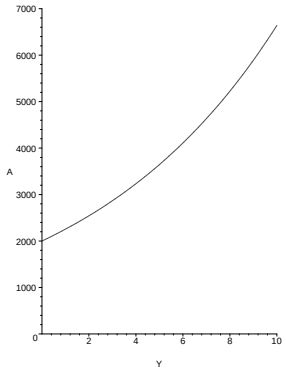
4. Let's go back to my friend Jenny. She's looking at \$2000 a sweet hedge fund that makes 12% APR, compounded continuously.

   (a) Make a table of values listing the amount of money in her account each year for the next 10 years.

   (b) Use your table of values to graph the function whose *independent* variable is the number of years passed, and *dependent* variable is the amount of money in her bank account.
   The function we're dealing with for both parts is:

$$A(Y) = \$2000 \cdot e^{0.12 \cdot Y}$$

Y	$A(Y)$
0	\$2000
1	\$2254.99
2	\$2542.49
3	\$2866.65
4	\$3232.14
5	\$3644.23
6	\$4108.86
7	\$4632.73
8	\$5223.39
9	\$5889.35
10	\$6640.23



   (c) What is the domain and range of this function?

   Domain: $[0, \infty)$ (all non-negative numbers, or $Y \geq 0$) – she deposits the money at year zero, and can take it out whenever she wants.

   Range: $[2000, \infty)$ (all numbers greater than or equal to 2000, or $A(Y) \geq 2000$) – she always has to have more than \$2000 in her account.

5. As great as the hedge fund seemed, Jenny is sensibly suspicious of anybody promising 12% APR with no risk. She starts looking at some other accounts instead. Suppose she is looking to deposit $2000.

   (a) Graph **on the same set of axes** the amount of money she'd have in her bank account over the next ten years in each of the following accounts:

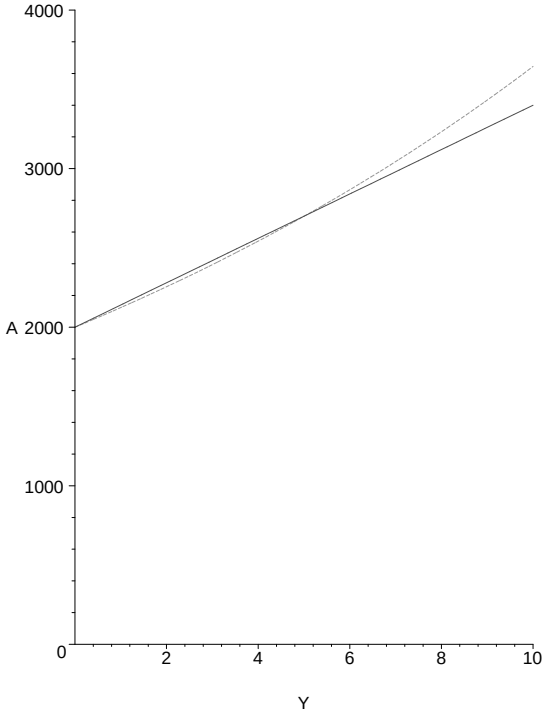
   - She is getting *simple* interest (p. 229, if you need a review), at 7% per year, paid annually

     $$A_1(Y) = 2000 + 0.07 \cdot 2000 \cdot Y$$

     This is the *solid* line on the graph below.
   - She is getting *compound* interest, at 6% APR, compounded continuously

     $$A_2(Y) = 2000 \cdot e^{0.06 \cdot Y}$$

     This is the *dashed* line on the graph below.



   (b) How long would it take for the amount of money in the second account to overtake the amount in the first account?

   The two graphs cross a little past five years. This is the point where the second account overtakes the first account.