Name:

Exam 2

- Do the following problems on the paper given.
- Show all work – the more work you show, the more partial credit you'll get!
- **Make sure you include units on all of your answers!**

Here are some formulas you might find helpful:

- Savings Plan Formula:

$$A = \text{PMT} \times \frac{\left[\left(1 + \frac{\text{APR}}{n}\right)^{(nY)} - 1\right]}{\left(\frac{\text{APR}}{n}\right)}$$

- Loan Payments Formula:

$$\text{PMT} = \frac{P \times \left(\frac{\text{APR}}{n}\right)}{\left[1 - \left(1 + \frac{\text{APR}}{n}\right)^{(-nY)}\right]}$$

1. Your friend Bobby has always wanted to be an astronaut, so he is looking at spaceship prices, even though he can't afford one. A Soyuz rocket costs \$20 million, while an air-launched moon rocket costs \$50 million.

   Given the following sentence:

   A Soyuz rocket costs 60 percent less than an air-launched moon rocket.

   (a) What is the reference value?

   \$50 million

   (b) What is the absolute difference?

   \$20 million − \$50 million = −\$30 million

   (c) Fill in the blanks.

   $$\frac{\$20 \text{ million} - \$50 \text{ million}}{\$50 \text{ million}} = -0.6 = -60\%.$$

2. Bobby is looking to open a savings account so that he can put himself through astronaut school. He will be depositing \$25,000.

   (a) One option is an account that makes 8% APR, compounded quarterly. How much will he have in 10 years?

   $$A = 25{,}000 \cdot \left(1 + \frac{0.08}{4}\right)^{4 \cdot 10}$$
   $$= \$55{,}200.99$$

(b) How much will he have after just one year? What is the APY?

$$A = \$25{,}000 \cdot \left(1 + \frac{0.08}{4}\right)^{4 \cdot 1}$$
$$= \boxed{\$27{,}060.80}$$

$$\text{APY} = \text{rel. change after 1 year}$$
$$= \frac{\$27{,}060.80 - \$25{,}000}{\$25{,}000}$$
$$= 0.0824$$
$$= \boxed{8.24\%}$$

3. Instead of having to wait years and save up lots of money to pay for astronaut school, Bobby has decided to take out a loan for $100,000.

(a) If he takes out a loan with a 20 year term and an APR of 3%, what will his monthly payments be?

$$\text{PMT} = \frac{\$100{,}000 \cdot \left(\frac{0.03}{12}\right)}{\left(1 - \left(1 + \frac{0.03}{12}\right)^{12 \cdot 20}\right)}$$
$$= \boxed{\$554.60}$$

(b) After he's made his student loan payments, Bobby still has $100 left over every month. He decides to start saving his extra money in an account that makes 5% APR. How much will he have after 15 years?

$$A = \$100 \cdot \frac{\left(\left(1 + \frac{0.05}{12}\right)^{12 \cdot 15} - 1\right)}{\left(\frac{0.05}{12}\right)}$$
$$= \boxed{\$26{,}728.89}$$

4. The manned spaceflight industry is in a rut right now, so Bobby is having trouble finding a job as an astronaut. To make some extra money on the side, he starts farming mealworms (a type of grub often used as bait and pet food).

   (a) The number of mealworms in Bobby's mealworm farm grows by 3% per day. How long will it take for their population to double?

$$T_{doub} = \frac{\log(2)}{\log(1+0.03)} = \boxed{23.45 \text{ days}}$$

OR

$$Q = Q_0 \cdot (1+r)^t$$
$$2 = 1 \cdot (1+0.03)^t$$
$$\log(2) = \log\left((1+0.03)^t\right)$$
$$\log(2) = t\log(1+0.03)$$
$$t = \frac{\log(2)}{\log(1+0.03)} = \boxed{23.45 \text{ days}}$$

   (b) Bobby just scored a huge mealworm contract with Bob's Bait Emporium! How many mealworms does he need to have now if he needs 100,000 mealworms in six months? (You can assume that a month is 30 days.) $6 \text{ months} \cdot \frac{30 \text{ days}}{1 \text{ mo}} = 180 \text{ days}$

$$100{,}000 = Q_0 \cdot (1+0.03)^{180}$$
$$\frac{\square}{\square}(1+0.03)^{180}$$
$$\frac{100{,}000}{(1+0.03)^{180}} = 488.989$$

$\boxed{\text{So 489 mealworms!}}$

OR

$$100{,}000 = Q_0 \cdot 2^{180/23.45}$$
$$\frac{100{,}000}{2^{(180/23.45)}} = Q_0$$
$$Q_0 = 489 \text{ mealworms}$$

5. Bobby's friends are so grossed out by his mealworm farm that they stop hanging out with him. He has 8% fewer friends every month. Right now he has 80 friends.

(a) How long will it be before he has 40 friends? How long before he has 10?

40 friends is half of 80.
We are looking for the half-life.

$$T_{half} = -\frac{\log(2)}{\log(1+r)}$$

$$= -\frac{\log(2)}{\log(1-0.08)} = \boxed{8.313 \text{ months}}$$

Second Part:

$$10 = 80 \cdot (1-0.08)^t$$

$$\frac{10}{80} = (1-0.08)^t$$

$$\log\left(\frac{10}{80}\right) = \log\left((1-0.08)^t\right)$$

$$\log\left(\frac{10}{80}\right) = t\log(1-0.08)$$

$$t = \frac{\log\left(\frac{10}{80}\right)}{\log(1-0.08)} = \boxed{24.939 \text{ months.}}$$

OR

40 is $\frac{1}{2}$ of 80
20 is $\frac{1}{2}$ of 40
10 is $\frac{1}{2}$ of 20
So 3 half lives!

$3 \cdot 8.313 = \boxed{24.939 \text{ months}}$

(b) Bobby can't get rid of his mealworm farm for another six months. How many friends will he have left then?

$$Q(6 \text{ months}) = 80 \cdot (1-0.08)^6$$
$$= 48.8$$

So 48 friends & 1 who is mildly disgusted.

OR

$$Q(6 \text{ months}) = 80 \cdot \left(\frac{1}{2}\right)^{6/8.313}$$
$$= 48.8 \text{ friends}$$

6. Bobby finally got a job as an astronaut! He is going to Mars and will be gone for 5 years. All of his friends come back, and give him a going-away present of $5,000. He puts it in a CD that makes 5% APR, compounded continuously. How much money will he have when he gets back?

$A = \$5{,}000 \cdot e^{0.05 \cdot 5}$
$= \boxed{\$6{,}420.12}$

7. It's the day of the rocket launch. Today Bobby will be flying from Florida to the International Space Station. The flight starts at 1:00PM at 0 miles above sea level, and ends at 3:00PM at 200 miles above sea level. Imagine the function describing Bobby's altitude at a given time.

   (a) What is the independent variable?

   Time

   (b) What is the dependent variable?

   Altitude

   (c) What is the domain of the function?

   1:00 – 3:00 PM

   (d) What is the range of the function?

   0 – 200 miles