Name: Friday, October 10

Exam 1

Show your work! I can't give you partial credit if I don't know what you were trying to do.

Make sure you show all of your units! I may take a point off if you leave units off of your answers.

Some formulas for you:

- Continuously Compounded Interest:

$$A = Pe^{(\text{APR}\cdot Y)} \text{ where } e \approx 2.71828183$$

- Savings Plan:

$$A = \text{PMT} \times \frac{\left[\left(1+\frac{\text{APR}}{n}\right)^{(nY)} - 1\right]}{\left(\frac{\text{APR}}{n}\right)}$$

- Loan Payments:

$$\text{PMT} = \frac{P \times \left(\frac{\text{APR}}{n}\right)}{\left[1 - \left(1+\frac{\text{APR}}{n}\right)^{(-nY)}\right]}$$

Good luck!

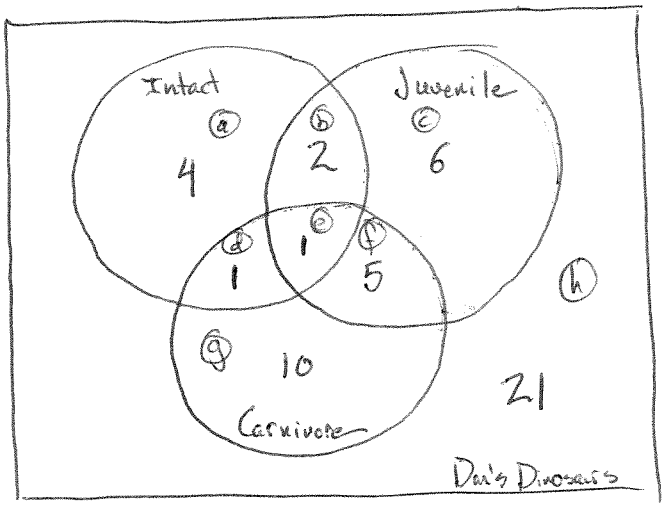
1	20
2	10
3	16
4	15
5	10
6	15
7	20
8	—
Total	100

Median: 81

1. Professor Dan is on a paleontology dig. They found 50 dinosaur skeletons. Some were intact, some were juveniles, and some were carnivores. Here's what they found:

   - 1 intact juvenile carnivore
   - 2 intact carnivores
   - 3 intact juveniles
   - 6 juvenile carnivores
   - 17 carnivores
   - 14 juveniles
   - 8 intact skeletons

   (a) Draw a Venn diagram illustrating this data.



   (b) How many skeletons were either juvenile or intact (or both)?

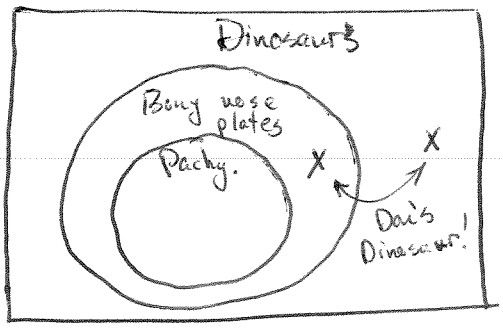
   $a + b + c + d + e + f = 4 + 2 + 6 + 1 + 1 + 5 =$ 19 skeletons

   (c) How many skeletons were not intact, juvenile, or carnivores?

   $h =$ 21 skeletons

2. Dan just discovered an interesting new skeleton. He's not sure what it is, though, and he thinks there might be a mistake in his dichotomous key. Determine if the following argument is valid, and use a Venn diagram to justify your answer.

   - Every dinosaur that is a pachyrhinosaurus has a bony nose plate.
   - Dan's dinosaur isn't a pachyrhinosaurus.
   - Dan's dinosaur doesn't have a bony nose plate.



Invalid.

3. Dan and his team have had to dig out 5000 cubic meters of shale. Shale weighs 2.2 grams per cubic centimeter. How many kilograms of sandstone did they move? (If you can't remember what the metric prefixes mean, take your best guess!)

$$5000\ m^3 \cdot \left(\frac{100\ cm}{1\ m}\right)^3 \cdot \left(\frac{2.2\ g}{cm^3}\right) \cdot \left(\frac{1\ kg}{1000\ g}\right) = 11{,}000{,}000\ kg$$

$$= 1.1 \cdot 10^7 \text{ kilograms}$$

4. While Dan and his team were excavating, they got bitten by a few bugs.

   (a) The team got bitten 40,000 times, and lost 0.0005 liters of blood each time. Rewrite both numbers in scientific notation, and then calculate the total amount of blood they lost. Write your answer in scientific notation.

$40{,}000 = 4 \cdot 10^4$ bites

$0.0005 = 5 \cdot 10^{-4} \frac{L}{\text{bite}}$

$4 \cdot 10^4 \text{ bites} \cdot 5 \cdot 10^{-4} \frac{L}{\text{bite}} = 4.5 \cdot 10^4 \cdot 10^{-4} = 20 \cdot 10^{4-4}$

$= 20 \cdot 10^0 = \boxed{2 \cdot 10^1 \text{ Liters}}$

   (b) Calculate $\frac{70{,}000{,}000{,}000}{5{,}000{,}000}$ by first converting each to scientific notation. Then write your answer in scientific notation.

$70{,}000{,}000{,}000 = 7 \cdot 10^{10}$

$5{,}000{,}000 = 5 \cdot 10^6$

$\frac{7 \cdot 10^{10}}{5 \cdot 10^6} = \frac{7}{5} \cdot 10^{10-6} = \boxed{1.4 \cdot 10^4}$

5. In 2007, Dan was given a travel grant for \$4000. Thanks to Dan's amazing work in British Columbia, his travel grant was increased by 50% from 2007 to 2008. But next year, because of a scandal that he'd rather not go into, his funding has been cut by 50% from 2008 to 2009. How much is his travel grant for 2009?

$\$4000 \cdot \left(1 + \frac{50}{100}\right)\left(1 - \frac{50}{100}\right) = \boxed{\$3000}$

increased by 50%

decreased by 50%

OR:

In 2007: \$4000

In 2008: $\$4000 + \frac{50}{100}\$4000 = \$4000 + \$2000 = \$6000$

In 2009: $\$6000 - \frac{50}{100}\$6000 = \$6000 - \$3000 = \boxed{\$3000}$

6. The Paleontology Society was so impressed by Professor Dan's work that it has awarded him the Schuchert Award, which comes with a $2,000 grant. He wants to deposit it in a bank account.

   (a) His bank is offering him an account at 4.1% APR, compounded continuously, or a CD at 4.15% APR compounded annually. Compute the APY for each account. Which is better?

Continuously: After 1 year: $2000 \cdot e^{0.041 \cdot 1} = 2083.7042$

APY $\frac{2083.7042 - 2000}{2000} = 0.04185 = 4.185\%$

Annually: $2000 \cdot (1 + 0.0415) = 2083$

$\frac{2083 - 2000}{2000} = 0.0415 = 4.15\%$

Continuously is better.

   (b) How ~~much would he have in each~~ account in 18 years?

Continuous: $A = 2000 \cdot e^{0.041 \cdot 18}$

$= \$4183.49$

Annual: $A = 2000 \cdot (1 + 0.0415)^{18}$

$= \$4168.11$

7. Since Dan's student Larry is off in the middle of nowhere in British Columbia at a dig, his cost of living is lower than it normally would be. He's saving about $400 per month.

   (a) He's going to be at the dig for three years, and he'd like to have a car while he's there. He has his eye on an old used Saab. If he puts his extra money toward monthly payments on a loan at 6% APR, how large a loan can he afford?

$$\$400 = \frac{P \times \left(\frac{0.06}{12}\right)}{\left[1 - \left(1 + \frac{0.06}{12}\right)^{-12 \cdot 3}\right]}$$

$$\frac{\$400}{\left(\frac{\left(\frac{0.06}{12}\right)}{\left[1 - \left(1 + \frac{0.06}{12}\right)^{(-12 \cdot 3)}\right]}\right)} = P$$

$$P = \$13,148.40$$

(b) Suppose instead he puts the \$400 per month in a money market account at 4% APR. How much will he have when he gets back?

$$A = \$400 \cdot \frac{\left[\left(1+\frac{0.04}{12}\right)^{(12\cdot 3)} - 1\right]}{\left(\frac{0.04}{12}\right)}$$

$$= \boxed{\$15{,}272.62}$$

(c) What percentage of that amount is from interest?

Amount paid: $\$400 \cdot 12 \cdot 3 = \$14{,}400.00$

Interest: $\$15{,}272.62 - \$14{,}400.00 = \$872.62$

% that was interest:

$$\frac{\$872.62}{\$15{,}272.62} = 0.05714$$

$$= \boxed{5.714\%}$$