

MATH 5720—HW 8

Due Thursday, March 26, 2009

1. Strang Problem 6.2.19
2. For the difference equation

$$u_{n+1} = u_n e^{r(1 - \frac{u_n}{K})}$$

Find all equilibria U , and determine the stability criteria for each equilibrium. You may assume that $r > 0$ and $K > 0$. Sketch this mapping in the (u_n, u_{n+1}) plane for two different values of r ($r = 0.5$ and $r = 1.5$), and perform an iteration by sketching a cobwebbing in each case.

3. Consider the difference equation

$$u_{n+1} = u_n \left(\frac{R_0}{1 + \frac{R_0 - 1}{K} u_n} \right)^\beta$$

with $\beta > 0$, $K > 0$, and $R_0 > 1$. Show that $U = K$ is an equilibrium, and find its stability criterion. In the (R_0, β) parameter plane, sketch the boundary that separates the region of stability and instability of $U = K$.