MATH 5720—HW 12

Due Thursday, April 23, 2009

1. Solve the following initial value problem for $u(x,t)$:

$$\frac{\partial u}{\partial t} + u^2 \frac{\partial u}{\partial x} = 0$$

with

$$u(x,0) = \begin{cases} 3 & x < 0 \\ 4 & x > 0 \end{cases}$$

2. Suppose that drivers respond to traffic density u by driving at a velocity v according to

$$v(u) = v_{max}\left(1 - \frac{u}{u_{max}}\right),$$

where the constant v_{max} is the maximum speed (with no cars nearby), and the constant u_{max} is the maximum traffic density (at which gridlock occurs).

Now consider a queue of cars of length L, lined up behind a red traffic light at maximum density. Assume there are no cars in front or in back of the length-L line.

In terms of v_{max}, u_{max}, and L, how long after the light turns green will it take for the last car in the queue to pass the traffic light?

Use this traffic flow equation:

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(uv) = 0$$

with an initial condition describing the above situation.