MATH 4200, INTRODUCTION TO COMPLEX VARIABLES, FALL 2018

Classroom:	JWB 333
Time:	MWF 11:50 – 12:40
Instructor:	Domingo Toledo
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Office Hours:	Mon 12:55–1:45, Wed 10:45 - 11:35, or by appointment.
Web-page:	`http://www.math.utah.edu/~toledo/4200F18.html`
Prerequisites:	"C" or better in Math 3220
Textbook:	Joseph L.Taylor *Complex Variables* Pure and Applied Undergraduate Texts 16 American Mathematical Society Print ISBN: 978-0-8218-6901-7, Electronic ISBN: 978-1-4704-1129-9 Can be ordered from `https://bookstore.ams.org/amstext-16/`

Course Description: This course is an introduction to the theory of complex-valued functions of a complex variable. We will be following Taylor's textbook, .covering Chapters 1,2,3 and parts of 4,5,6..

We will begin in Chapter 1 by defining the field of complex numbers $\mathbb{C} = \{z = x + iy \ : \ x, y \in \mathbb{R}\}$ with natural addition and multiplication for which $i^2 = -1$. As a real vector space $\mathbb{C}$ is isomorphic to $\mathbb{R}^2$.

The purpose of the course is to study functions $f : \mathbb{C} \to \mathbb{C}$ (more generally $f : U \to \mathbb{C}$ for some open subset U of $\mathbb{C}$) which are *differentiable in the complex sense*, meaning that for all $z \in U$ and *for all* $h \in \mathbb{C}$ sufficiently small but $\neq 0$, the limit of the usual difference quotient defining the derivative $f'(z)$ exists:

$$f'(z) = \lim_{h\to 0} \frac{f(z+h) - f(z)}{h} \quad \text{exists}$$

The class of functions satisfying this definition turns out to have many surprising properties and many applications. To mention a few:

- If f is differentiable in the complex sense, then it has derivatives of al orders (in sharp contrast with the differentiable functions $U \to \mathbb{R}^2$ that need not have derivatives of order > 1)

- *Even more*: If f is differentiable in the complex sense, then, about any z_0 in its domain, f has a convergent power series expansion

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

convergent in some disk $\{|z - z_0| < \epsilon\}$ centered at z_0. For this reason the functions we are studying are usually called *analytic functions* of a complex variable. This is again in sharp contrast with functions of a real variable, where even having infinitely many derivatives does not guarantee a convergent power series expansion.
- Power series expansions are deduced in Chapter 3 from *Cauchy's integral formula*. This formula is the most important ingredient in the course. It is discussed and proved in Chapter 2.
- Connections with partial differential equations: If f is an analytic function, its real and imaginary parts u, v defined by $f(x + iy) = u(x, y) + iv(x, y)$ satisfy

$$u_x = v_y \quad \text{and} \quad u_y = -v_x$$

(subscripts mean partial derivatives) called the Cauchy-Riemann equations. The functions u, v then satisfy Laplace's equation:

$$u_{xx} + u_{yy} = 0, \quad \text{similarly} \quad v_{xx} + v_{yy} = 0,$$

in other words, u and v are *harmonic functions*.
- A *Conformal map* $f : U \to \mathbb{C}$ means a map that preserves angles. Analytic functions are conformal maps at all points z where $f'(z) \neq 0$
- Many other applications, for example, computing definite integrals by residues.
- We hope to have some time at the end of the semester to look briefly at some special topics, for example, the Riemann zeta-function, prime numbers, the Riemann hypothesis.

Proofs This is a rigorous mathematics course. You will be expected to follow proofs from the textbook and the lectures. You will also be expected to produce proofs in the homework and the exams. The level of the proofs will be as in the Foundations of Analysis series, Math 3210-3220. This means that you need to understand as well as write proofs of existence of limits, of continuity of functions, of properties of subsets of the plane (open, closed, compact, connected, etc), and other proofs that come up in analysis. These proofs often involve both logic and inequalities (estimates).

Lecture Notes My lectures in class will be projected on a screen. At the end of each week I will post pdf files of these projections. Keep in mind that everything you see projected during class will be available to you at the end of the week.

Homework: I will be assigning homework problems to be collected roughly every other week. The problems will be of two types: computational and theoretical, and you will need to become proficient in both. For theoretical problems you will usually need to give proofs, written in correct logical form. This means that you must be comfortable with the process of writing proofs.

Exams: There will be two midterm exams on **September 26** and **November 7**, and a comprehensive final exam on **Thursday, December 13, 10:30–12:30**. Please save these dates!

Grading:	Homework, drop lowest 2:	35 %
	Midterm Exams:	40 %
	Final Exam:	25 %

Important dates:

- Classes begin: Monday, August 20.
- Last day to add classes without a permission code: Friday, August 24.
- Last day to wait list: Friday, August 24.
- Last day to add, drop (delete), elect CR/NC, or audit classes: Friday, August 31.
- Tuition payment due: Friday, August 31..
- Last day to withdraw from classes: Friday, October 19..
- Last day to reverse CR/NC option: Friday, November 30..
- Classes end: Thursday, December 6.
- Reading day: Friday December 7.
- Final Exam Period: Monday-Friday December 10–14.

Holidays

- Labor Day: Monday, September 3.
- Fall Break: Sunday - Sunday, October 7 – 14.
- Thanksgiving Break: Thursday-Sunday, November 22–25.

ADA: The Americans with Disabilities Act requires that reasonable accommodations be provided for students with physical, cognitive, systemic, learning, and psychiatric disabilities. Please contact me at the beginning of the semester to discuss any such accommodations you may require for this course.