The final exam will be comprehensive. You should know the material covered in the first two midterms. For the earlier material, as well as the new, you should:

- Know the definitions of all important terms.
- Know the statements of all important theorems, and how to prove them (for shorter proofs) or some of the ideas of the proof (for longer proofs).
- Be able to do simple problems.

For the older material, refer to the review sheets for the midterms. For the newer material, see below. *Note*: I have posted a revised version of the notes, Version 1. **All references given here are to Version 1**. The references in the reviews for the two midterms are to Version 0, which is still posted. There should be no conflict in the numbering for the older material in the midterms. The only substantial changes in Version 1 are to the last two sections of the notes.

1. Know the definitions of all the following terms:
   (a) Connected components of a topological space N5.1, Def 5.21
   (b) Smooth function on open sets in $\mathbb{R}^n$ N6, Remark 6.2, (4).
   (c) Topological surface N6, Def 6.1
   (d) Atlas, charts, transition functions, smooth surface. N6 Def 6.1
   (e) Smooth function on a smooth surface. N6 Def 6.3
   (f) Length of curves on a smooth surface N 6.2.1
   (g) Intrinsic metric on a surface N 6.2
   (h) Polar coordinates, spherical coordinates N 6.2 Examples 6.15. 6.16
   (i) Geodesic Curvature N 6.3.2 Proof of Thm 6.20
   (j) Geodesics N 6.3.2, Defs 6.22, 6.24
   (k) Exponential map, N 6.3.4 Def 6.28
   (l) Normal coordinates, geodesic polar coordinates N 6.3.4 Def 6.29.
   (m) Geodesic rays, geodesic circles N 6.3.4 Def 6.31 or N 6.4 Def 6.34
   (n) Gaussian curvature N 6.4 Def 6.37

2. Know the following theorems and have some idea of the proof:
   (a) Intermediate value theorem N 5.3, Thm 5.34
   (b) Implicit function theorem. N 5.3, Thm 5.35

(c) First variation formula. N 6.3.1

(d) Gauss lemma N 6.3.4 Thm 6.30

(e) Geodesic rays in a geodesic polar coordinate system have smallest length among all curves between their endpoints N 6.3.4 Thm 6.32

3. Know how to solve problems of the following nature:

   (a) Use connected components to distinguish spaces. For example, $\mathbb{R}$ not homeomorphic to $\mathbb{R}^2$, the space that looks like the letter X is not homeomorphic to the space that looks like the letter Y. N 5.1

   (b) Use connectedness to prove the intermediate value theorem, the implicit function theorem. N 5.3

   (c) Show that a space is a smooth surface by finding an atlas and checking that the transition functions are smooth. For example show how S^2 is a smooth surface by

      i. Applying the inverse function theorem to the defining equation $x^2 + y^2 + z^2 - 1 = 0$ N 6, Example 6.8

      ii. By stereographic projection (Homework 6)

      iii. By using spherical coordinates.

      Or show that the torus T is a smooth surface by using the defining identifications on a square to give it an atlas. N6 Example 6.10 or Homework 6

   (d) Show that certain curves are geodesics by checking the geodesic equation, or being able to compute geodesic curvature. N 6.3.2 Example 6.21, Homework 7.

   (e) Show certain curves are geodesics by showing that they are absolute minimizers. N 6.2.2