For the midterm:

- Know the definitions of all important terms and statements of theorems. (listed below).
- Be able to do *simple* problems, such as verifications of examples, or proofs of theorems that have simple proofs.

Note: References in parenthesis are to the posted notes (N, followed by section number), or to the posted weekly lectures (W, followed by week number).

1. Know the definitions of all the following:(N3.2, N3.3 , W5)
   (a) A topology on a set X.
   (b) A topological space.
   (c) Open set in a topological space, closed set in a topological space.
   (d) Continuous map $f : (X, \mathcal{T}_X) \to (Y, \mathcal{T}_Y)$ of topological spaces.
   (e) Homeomorphism, open map, closed map. (N4.3,def4.15 for open, closed map)
   (f) Interior E^0 and closure $\overline{E}$ of a subset E of a topological space.
   (g) Hausdorff space.
2. Know examples (and non-examples, such as non-Hausdorff spaces) of all of the above, and simple proofs, such as how to prove that a metric space is Hausdorff.
3. Know the definitions of all the following: (N3.4, W5, W6)
   (a) Basis of a topology.
   (b) Topology of a metric space (X, d).
   (c) Product topology on $X \times Y$ where $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ are topological spaces.
   (d) Product $\prod_{\alpha \in A} X_\alpha$ where $\{X_\alpha\}_{\alpha \in A}$ is an arbitrary collection of sets.
   (e) If the X_α are all topological spaces $(X_\alpha, \mathcal{T}_\alpha)$, definition of the product topology on $\prod_{\alpha \in A} X_\alpha$.
4. Know how to do the following: (N3.4, N3.5, W6, W7)
   (a) Given a collection $\mathcal{B}$ of subsets of a set X (that is, $\mathcal{B} \subset 2^X$), how to check that $\mathcal{B}$ is a basis for a topology on X.
   (b) Know how to apply the above condition to check that $\mathcal{B} = \{B(x, r) \mid x \in X, r > 0\}$ is a basis for a topology of a metric space (X, d).
   (c) Know how to check that $\mathcal{B} = \{U \times V \mid U \in \mathcal{T}_X, V \in \mathcal{T}_Y\}$ is a basis for a topology on $X \times Y$.

(d) Suppose that B and $\mathcal{B}'$ are two collections of subsets of X (that is $\mathcal{B}, \mathcal{B}' \subset 2^X$) that satisfy the conditions that guarantee that each is the basis of a topology on X, know how to check that B and B' generate the same topology on X.

(e) Know how to prove: A map $f : Z \to X \times Y$ is continuous ($X \times Y$ with the product topology) if and only if both compositions $p_X \circ f$ and $p_Y \circ f$ are continuous (where p_X, p_Y are the projections of $X \times Y$ to X, Y respectively.

(f) Same for infinite products $\prod_{\alpha \in Z} X_\alpha$.

5. Know hos to define (N4.1, N4.3, W7, W9)
   (a) Subspace topology (for $Y \subset X$).
   (b) Quotient topology (for : $X \to Y$ surjective)
   (c) $f : (X, \mathcal{T}_X) \to (Y, \mathcal{T}_Y)$ an identification.

6. Know how to prove: (N4.3, W9)
   (a) Sufficient conditions for identification: f open map, f closed map.
   (b) If $f : X \to Y$ is an identification, and $g : Y \to Z$, then g is continuous if and only if $g \circ f$ is continuous.
   (c) Relation with equivalence relations, for example, periodic functions.

7. Know the following definitions: (N4.2, N5.1,W7,W8, W9)
   (a) Compact topological space.
   (b) Connected topological space.
   (c) Path connected topological space.
   (d) A topological space is locally P, where P is a topological property, for example, connected.

8. Know how to prove:
   (a) Compact subspace of a metric space is bounded.
   (b) Closed subspace of a compact space is compact.
   (c) Compact subspace of a Hausdorff space is closed.
   (d) Example that shoes that Hausdorff is needed in last statement.
   (e) Continuous image of a compact space is compact.
   (f) Continuous map of a compact space to a Hausdorff space is a closed map.
   (g) Continuous bijection of a compact space to a Hausdorff space is a homeomorphism.
   (h) Continuous image of a connected space is connected.
   (i) Path connected space is connected.
   (j) Connected, locally path connected space is path connected.
   (k) Convex subsets of $\mathbb{R}^n$ are connected.