References are to Herstein's book (H) or Milne's notes (M), or homework (HW).

Definitions

1. Ring, commutative ring, integral domain, division ring, field (H 4.1).
2. Ideal, maximal ideal, ring homomophism, kernel, quotient ring (H 4.3–4.4).
3. Polynomial ring, irreducible polynomial, greatest common divisor (H 4.5).
4. Field extension, degree of a field extension, finite extension (H 5.3–5.4).
5. Algebraic and transcendental elements, minimal polynomial of an algebraic element, degree of an algebraic element (H 5.3–5.4).
6. Characteristic of a field (H 5.1).
7. Constructible numbers (H 5.5).
8. Splitting field of a polynomial (M2).
9. F-homomorphism $\phi : E \to E'$ where E, E' are field extensions of F (M2).
10. F-automorphism of a field extension $E \supset F$ (M3).
11. Galois extension $E \supset F$, Galois group $Gal(E/F)$ (M3.9).
12. Discriminant of a polynomial (HW8, also M4).

Theorems

1. Basic theorems on rings and fields from Herstein (material of first two midterms).
2. If $L \supset K \supset F$ are field extensions, then $[L : F] = [L : K][K : F]$ (H 5.3.1).
3. If $K \supset F$ is a finite extension, any $\alpha \in K$ is algebraic of degree at most $[K : F]$ (Midterm 2).
4. $\sqrt[3]{2}$ is not constructible (H 5.5).
5. Construction of F-homomorphisms (M 2.1–2.2).
6. Existence and uniqueness of splitting fields (M 2.4, 2.7).
7. A finite extension $K \supset F$ is Galois if and only if it is the splitting field of a separable polynomial $f(X) \in F[X]$ (M3.10).
8. If $K \supset F$ is Galois, then $Gal(E/F)$ has order $[E : F]$. (M 3.2 and above equivalence) If $K \supset F$ is any finite extension, then the order of $Aut(E/F)$ is at most $[E : F]$ (consequence of M 2.8).

9. Fundamental Theorem of Galois Theory (M3.16): Know how to use it.

Problems

1. Know how to use that if $K \supset F$ is a field extension, then K is a vector space over F. For example, how to use this to find the cardinality of finite fields (HW 7), or to prove that certain numbers are algebraic (HW 6, 7).
2. Know how to use the multiplicativity of degrees (item 2 in the list of Theorems) (HW 7, Review problems for midterm 2).
3. Be able to give examples of extensions $K \supset F$ that are Galois, and examples of extensions that are not.
4. Be able to find splitting fields (HW 8, 9).
5. Be able to compute Galois groups in simple situations, and to apply the main theorem to find intermediate fields (HW 9).