

1. Find the splitting fields of the following polynomials in  $\mathbb{Q}[X]$ . In each case find the Galois group and find all the intermediate fields (fields between  $\mathbb{Q}$  and the splitting field).

(a)  $(X^2 - 2X - 1)(X^2 - 2X - 7)$ .

(b)  $(X^2 - 2X - 1)(X^2 - 2X - 5)$ .

(c)  $X^4 + 1$ .

2. Let  $K = \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$ .

(a) Find  $[K : \mathbb{Q}]$  and prove that  $K$  is Galois over  $\mathbb{Q}$ .

(b) Determine the Galois group of  $K$  over  $\mathbb{Q}$  and list all the intermediate fields  $L$ ,  $\mathbb{Q} \subset L \subset K$ .