Solutions to Selected Exam Practice Problems

(2) Done in class

(3) Since $\vec{u}$ and $\vec{v}$ are unit vectors, $\|\vec{u}\|=1$ and $\|\vec{v}\|=1$.

The Cauchy-Schwarz inequality states

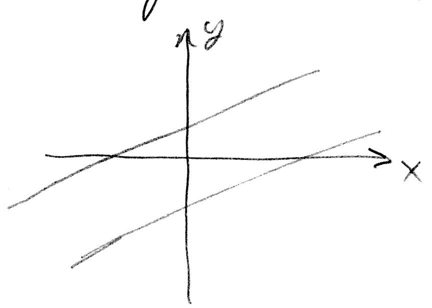
$$|\vec{u}\cdot\vec{v}| \leq \|\vec{u}\|\|\vec{v}\|$$

In this case, $|\vec{u}\cdot\vec{v}| \leq (1)(1) = 1$, so $\boxed{\text{NO}}$ $\vec{u}\cdot\vec{v}$ cannot ever equal 2.

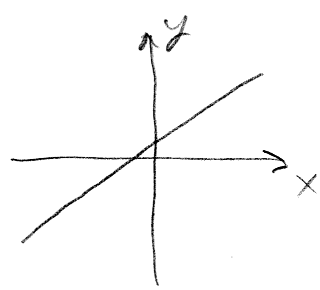
(4) Done in class.

(5) $(R^T R)^T = R^T (R^T)^T = R^T R$, so $R^T R$ is symmetric.

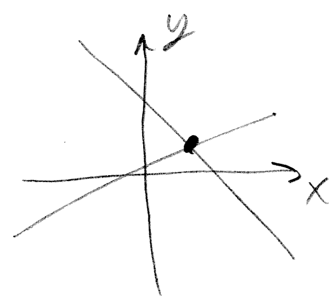
(6) No solution (the lines are parallel, so they never cross)



Infinitely many solutions (the lines are the same)



One solution



(7), (8) Not covered on exam.

(9) Done in class.

(10) ① The only solution to $A\vec{x}=\vec{0}$ is $\vec{x}=\vec{0}$.

② We can find n pivots using Gaussian elimination.

(11) Not covered on exam.

Note regarding 4b.

I apologize that I made a mistake in class regarding this problem. The question asks us to prove that if A is invertible, then its inverse is unique. There are many ways of proving this, and the proof I presented in class was correct. However, here is another possible proof:

Suppose B and C are both inverses of A. Then, by definition

$$AB = I \qquad AC = I$$
$$BA = I \qquad CA = I.$$

In particular, $BA = CA$.
We can then multiply both sides on the right by B (or C) to obtain

$$(BA)B = (CA)B$$

By the associative law, this is

$$\begin{aligned} (BA)B &= C(AB) \\ IB &= CI \\ B &= C. \quad \square \end{aligned}$$